AL.2,2008-278
University of Alberta Library
0 1620 3409970 3

MATHEMATICS

Equations and Inequations

Distance Learning

Alberta EDUCATION

Module 3

2 3 6 8 10

Mathematics 9

Module 3

EQUATIONS AND INEQUATIONS

Alberta
EDUCATION

Mathematics 9
Student Module Booklet
Module 3
Equations and Inequations
Learning Technologies Branch
ISBN 0-7741-1127-5

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

The Learning Technologies Branch has an Internet site that you may find useful. The address is as follows:

http://ednet.edc.gov.ab.ca/ltb

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

ALL RIGHTS RESERVED

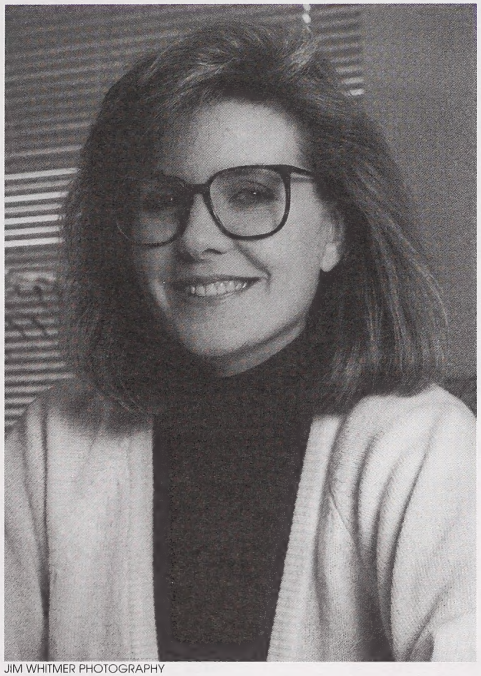
Copyright © 1997, the Crown in Right of Alberta, as represented by the Minister of Education, Alberta Education, 11160 Jasper Avenue, Edmonton, Alberta T5K 0L2. All rights reserved. Additional copies may be obtained from the Learning Resources Distributing Centre.

No part of this courseware may be reproduced in any form, including photocopying (unless otherwise indicated), without the written permission of Alberta Education.

Every effort has been made both to provide proper acknowledgement of the original source and to comply with copyright law. If cases are identified where this effort has been unsuccessful, please notify Alberta Education so that appropriate corrective action can be taken.

IT IS STRICTLY PROHIBITED TO COPY ANY PART OF THESE MATERIALS UNDER THE TERMS OF A LICENCE FROM A COLLECTIVE OR A LICENSING BODY.

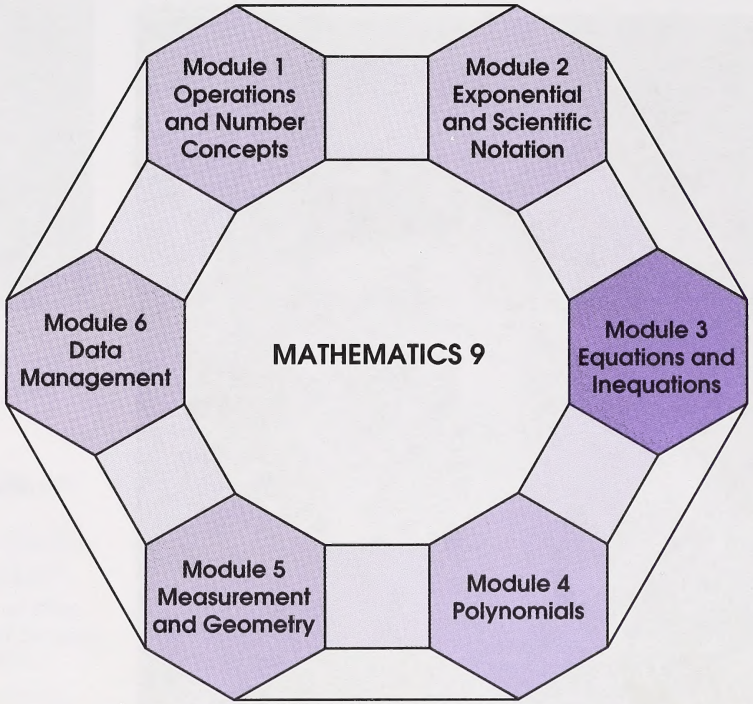
Welcome



JIM WHITMER PHOTOGRAPHY

Welcome to Module 3. We hope you'll enjoy your study of Equations and Inequations.

Mathematics 9 contains six modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.



The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

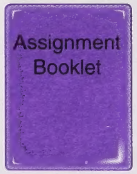
- Use computer software.

- Use a scientific calculator.

- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.

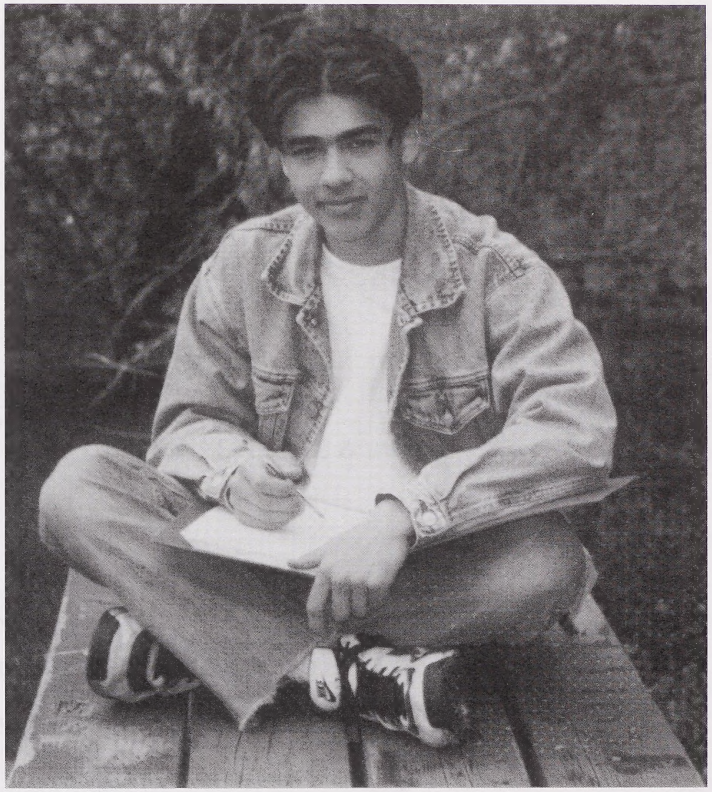


PHOTO SEARCH LTD.

There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

UNIVERSITY LIBRARY
UNIVERSITY OF ALBERTA

Technology

Today society is turning to **technology** more than ever before, and it is to your advantage to be able to effectively use technology when required.

Technology is the application of tools, materials, and processes to the solution of problems. More specifically, technology refers to devices and systems that are used in processing, transferring, storing, and communicating information through electronic media.

In Mathematics 9, along with the course materials, you will use a calculator, computer, and videocassette player as tools for learning and doing mathematics.

Calculators are helpful tools for solving problems and exploring patterns and relationships between numbers. Using a calculator will also save you time and help you develop your estimating skills. Therefore, you will be given numerous opportunities in each module to use a calculator.

Computers are useful for organizing and displaying data, or drawing figures. For this reason you will have the chance in many activities to work with popular computer applications such as spreadsheets and draw programs. You will also want to check out the many Internet connections in each module.

Videocassette players allow you to view video programs on key concepts that are difficult to explain in print. That is why video programs are cited in this course.

It is expected that all students will be able to view the video programs and use a calculator, and that most students will do the computer activities. However, if you are unable to access a computer, you may do the calculations using a calculator or draw figures and graphs by hand.

JIM WHITMER PHOTOGRAPHY

Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.

A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge.

The icon is a cue that the problem will be related to the topic of the activity.

The icon is a cue that the problem will provide a change of topic.

The Four-Stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem. You may consider the following strategies:

- changing your point of view
- using objects
- using diagrams
- making an organized list
- using Venn diagrams
- making a table
- guessing, checking, and revising
- acting out a problem
- working backwards
- simplifying a problem
- finding and applying a pattern
- using elimination
- using truth tables
- using an equation

Note: The Appendix in this module explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to this Appendix and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

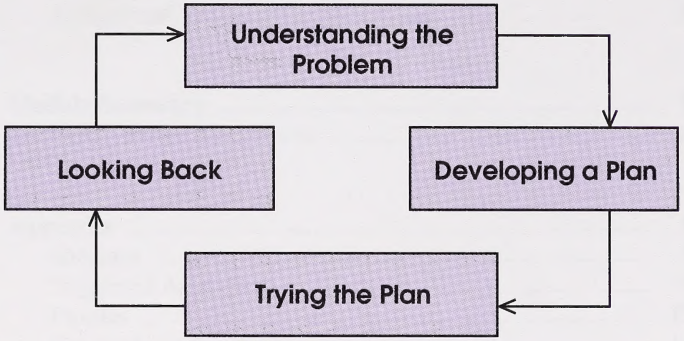
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

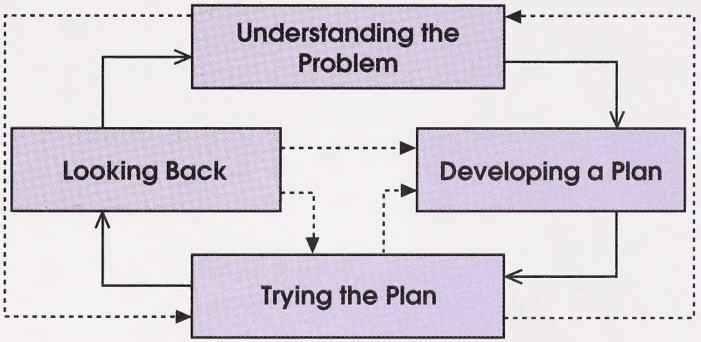
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



Digitized by the Internet Archive
in 2016 with funding from
University of Alberta Libraries

https://archive.org/details/mathematics9lear03albe

CONTENTS

Module Overview 1
Evaluation 2

Section 1: Equations 3
Activity 1: Solving Equations with Variables on One Side 4
Activity 2: Solving Equations with Variables on Both Sides 23
Activity 3: Solving Equations Containing Fractions 37
Follow-up Activities 45
Extra Help 45
Enrichment 51
Conclusion 58
Assignment 58

Section 2: Inequations 59
Activity 1: Modelling and Interpreting Inequations 60
Activity 2: Solving Inequations Involving Variables with a Coefficient of One 65
Activity 3: Solving Inequations Involving Variables with Coefficients Other than One 69
Activity 4: Solving Multi-Step Inequations 77
Follow-up Activities 80
Extra Help 80
Enrichment 85
Conclusion 87
Assignment 87

Module Summary 88
Final Module Assignment 88

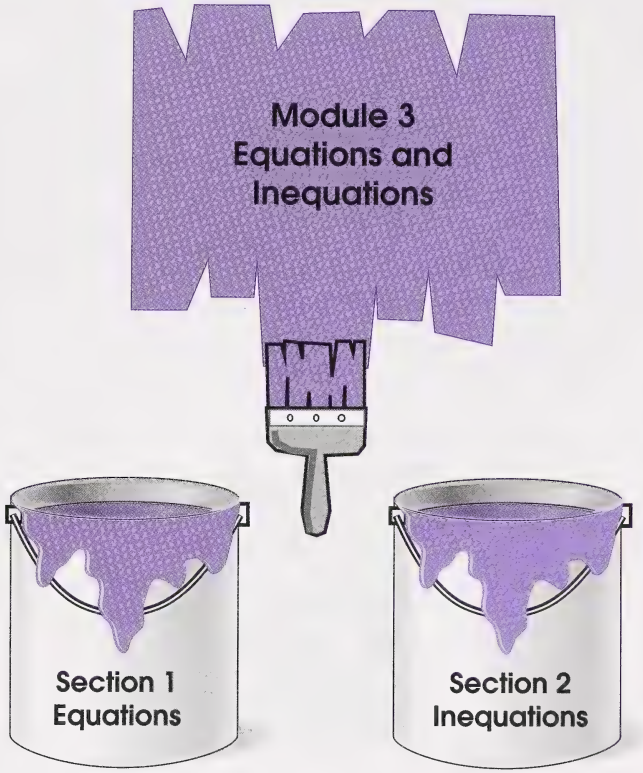
Appendix 89
Glossary 90
Suggested Answers 90
Puzzles 139
Cut-out Learning Aids 145

Module Overview

Think of a number between 1 and 10. Add 8 to it. Multiply the result by 2. Subtract 2. Divide by 2. Subtract the number that you first thought of. Is the answer 7? Try starting with a different number. Is the answer still 7? Why does this happen?

You could set up a table and look at the patterns that develop. Although you may find the patterns to be interesting, they do not reveal why the same answer always appears. If you use a symbol to represent the original number, you can discover the secret. The use of symbols to represent numbers and the arithmetic done involving these symbols provide the basis of algebra. You have been doing some form of algebra in your study of mathematics for one or two years now.

In this module you will extend your knowledge of algebra as you examine how to manipulate more complicated mathematical sentences (equations and inequations) and use them to help solve problems.

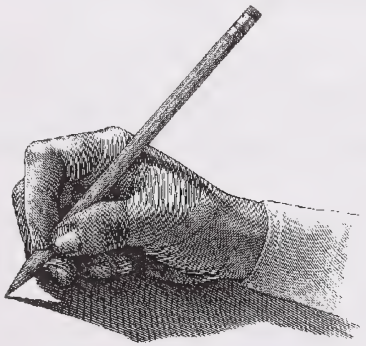


Evaluation

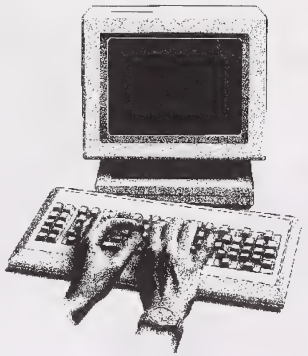
Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete two section assignments and a final module assignment. The mark distribution is as follows:

Section 1 Assignment	**45 marks**
Section 2 Assignment	**30 marks**
Final Module Assignment	**25 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.



If you are working on a CML terminal, you will have a module test as well as a module assignment.



Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

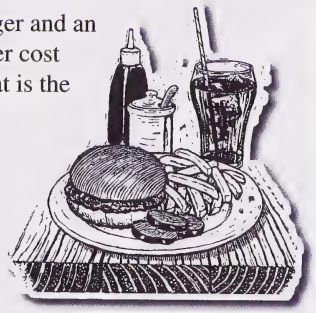
Section 1: Equations

Are you able to communicate using sign language? If so, then what are the two people in the photograph saying to each other? Sign language is a system of communication where hand gestures are used to represent words, ideas, and so on. If you do not know what the hand gestures represent, then you will not be able to understand what is being said.

As in sign language, if you do not understand what certain mathematical symbols or combinations of symbols and numbers mean, then you will not be able to understand what is being done or determine how to represent mathematical situations properly.

In this section you will use mathematical symbols and numbers to create mathematical sentences or equations. You will discover ways to manipulate mathematical sentences or equations to solve for a missing number. You will also discover how to create a problem, given the mathematical sentence.

Activity 1: Solving Equations with Variables on One Side



Geraldine spent \$5.60 on a cheeseburger and an order of french fries. The cheeseburger cost \$1.90 more than the french fries. What is the cost of each item?

A problem such as this requires an equation to solve it. In previous mathematics courses you solved and verified first-degree equations such as $5a+1=11$ and $2y+5=-3$. These equations were relatively easy to solve. You also used learning aids (such as an equation scale, counters, and cylinders) to solve and verify equations. In this section, the solutions to the equations you will encounter are somewhat more complicated. You will continue to use counters and cylinders to model equations. Other learning aids, such as algebra tiles, may also be used.

To start this activity, you will discover how to solve equations involving several steps by watching a video. You will then model (using counters and cylinders) and solve (using paper and pencil) examples of first-degree equations with variables on one side.

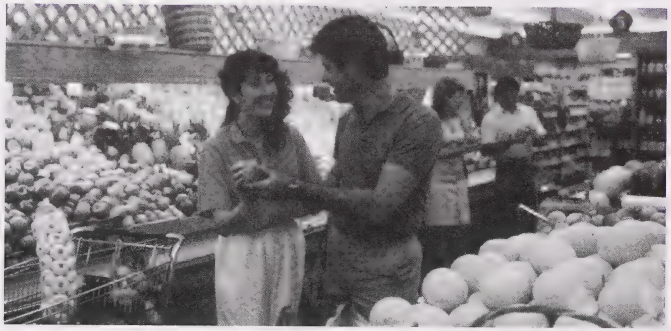
View the video *Equations—Solving with More than One Step* from the *Math Moves* series, ACCESS. Watch the part entitled "Solving Equations: $ax+bx=c$." Use the cut-out scale, counters, and cylinders from the Appendix to help you do the video assignment. This video is available from the Learning Resources Distributing Centre.

Check your answers to the video assignment by viewing the video.

1. How are the equations in the examples and questions on the video different from an equation such as $5x+3=11$?

Check your answer by turning to the Appendix.

Collecting Like Terms



Stores organize their shelves by putting like items together. This makes it easier for customers to find the items they need.

Simplifying equations with **like terms** makes them easier to solve.

Like terms are mathematical expressions that contain the same variable(s) with the same exponent.

Equations such as $3x + 6x = -18$ require you to collect like terms before solving.

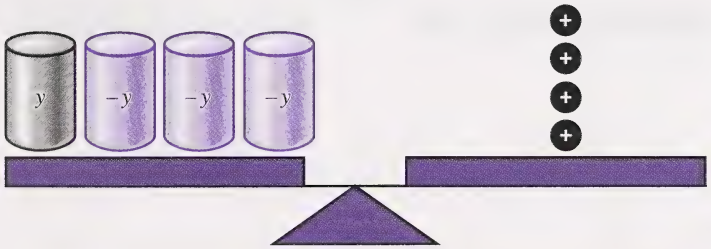
Now that you have been shown how to model equations of the form $ax + bx = c$ on the video, study the following examples in print. Use your cut-out learning aids to follow along. **Note:** For the purpose of photocopying, the negative cylinders and counters are white rather than coloured (as in the examples).

Example 1

Solve the equation $y - 3y = 4$.

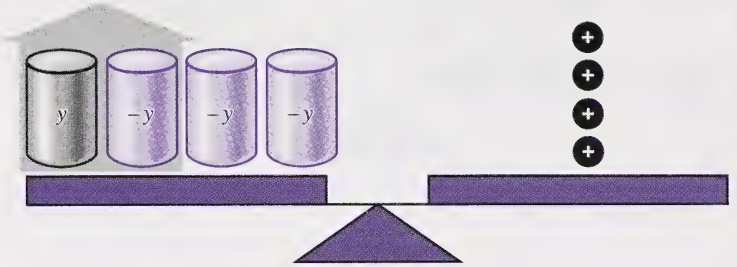
Solution

First model the equation.

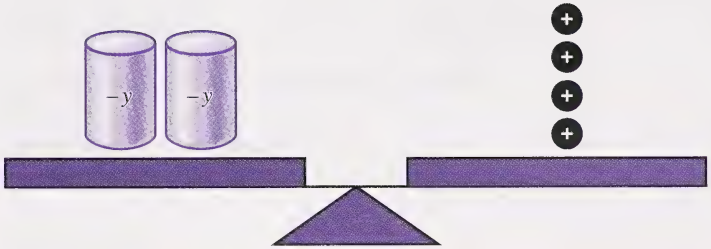


You know that a positive and a negative quantity result in zero. In this case you have one pair of cylinders that forms a zero (y and $-y$).

Simplify the equation by removing this zero.



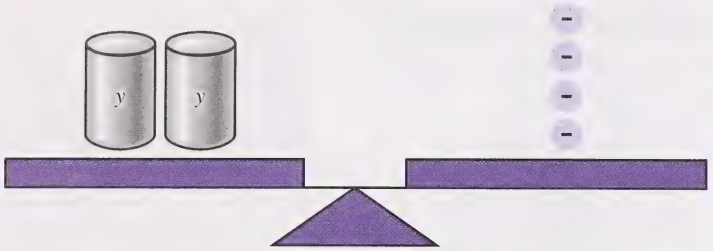
The result is this.



If you were to write the equivalent algebraic statement for this model, what would you write?

Remember that equations are balanced—the left side equals the right side. So, the statement will contain an equal sign. The left side shows $-2y$, and the right side shows 4. Therefore, the equivalent algebraic statement is $-2y = 4$.

Now complete the next step of your modelling by replacing each side of the equation with its inverse. This ensures that the variable you are solving for will be positive.

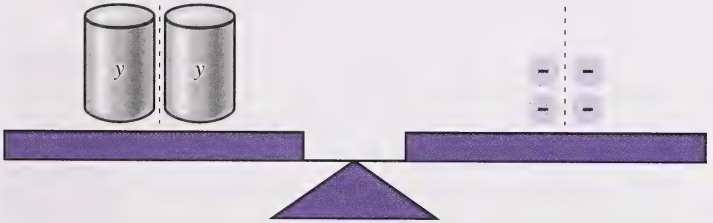


What would the equivalent algebraic statement for this model look like now?

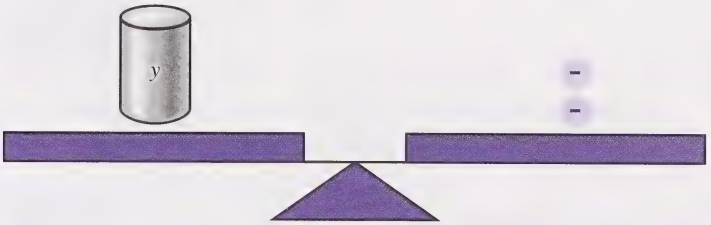
The equivalent algebraic statement for the previous model is $2y = -4$.

Note that you obtain $2y = -4$ algebraically by multiplying each side of $-2y = 4$ by -1.

In order to isolate the variable, divide each side into two groups.



To solve the equation, examine only one group from each side.



The solution is $y = -2$.

To complete this last step algebraically, divide each side of the equation by 2. Therefore, $y = -2$.

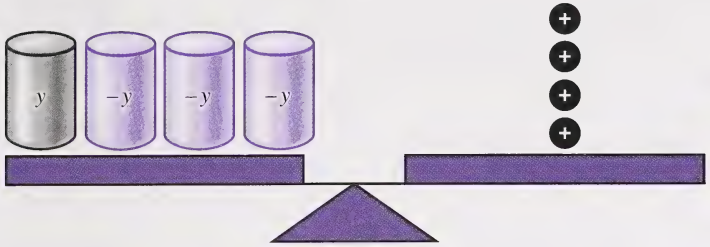
Here are all the algebraic steps gathered together.

$$y - 3y = 4 \qquad \text{← Combine like terms.}$$
$$-2y = 4$$
$$-2y \times (-1) = 4 \times (-1) \qquad \text{← Multiply both sides by – 1.}$$
$$2y = -4$$
$$\frac{2y}{2} = \frac{-4}{2} \qquad \text{← Divide both sides by 2.}$$
$$y = -2$$

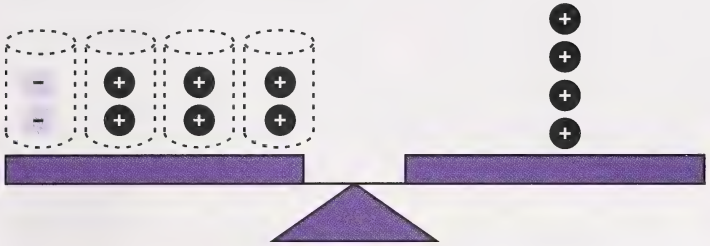
Instead of multiplying by -1 and dividing by 2, you can simply divide both sides by -2.

Verification

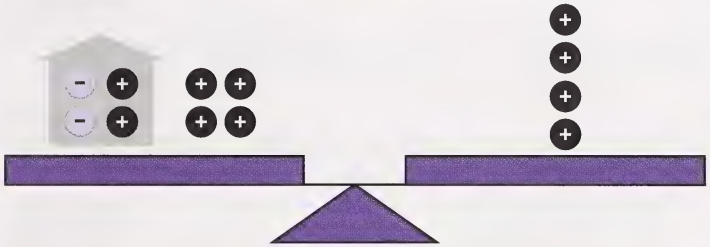
To verify the solution of $y - 3y = 4$, first model the equation.



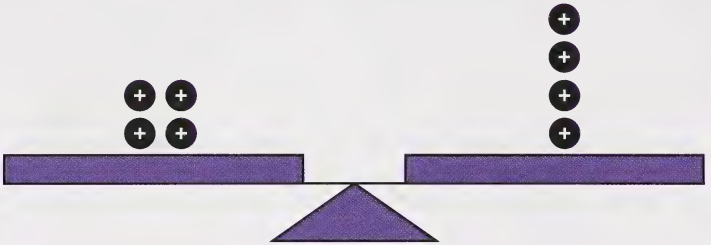
If $y = -2$, then $-y = 2$. Replace y with -2 and $-y$ with 2.



Remove the zeros.



This is the result.



Both sides have a value of $+4$ when $y = -2$; thus, the scale is balanced, and the solution is $y = -2$.

The following is the algebraic verification of Example 1.

Verification

	LS	RS
$y - 3y$		4
$= (-2) - 3(-2)$		
$= -2 + 6$		
$= 4$		
	LS =	RS

The left side is equal to the right side; thus, the solution is $y = -2$.

Do you remember the reason why a balance is used as a model when solving equations? A balance shows that both sides of an equation are equal. In order to keep an equation (or scale) balanced, you must do the required operation to **both** sides of the equation. Also, keep in mind that you must isolate the variable on one side of the equation.

2. Which step is equivalent to multiplying by -1 when modelling?

Check your answer by turning to the Appendix.

Example 2 is solved using paper and pencil. You may wish to model the solution using the cut-out learning aids.

Example 2

Solve the equation $5b-2b=9$.

Solution

$5b-2b=9$ ← Combine like terms.

$3b=9$

$\frac{3b}{3}=\frac{9}{3}$ ← Divide both sides by 3 to isolate b.

$b=3$

or

$5b-2b=9$ ← Combine like terms.

$3b=9$

$\frac{1}{\cancel{3}}b\times\frac{1}{\cancel{3}}=\frac{\cancel{9}^{3}}{1}\times\frac{1}{\cancel{3}}$ ← Multiply by the multiplicative inverse of 3.

$b=3$

The solution is $b=3$.

Verification

LS	RS
$5b-2b$	9
$=5(3)-2(3)$	
$=15-6$	
$=9$	

LS = RS

The left side is equal to the right side; thus, the solution is $b=3$.

Some equations you may need to solve may involve decimal numbers. While modelling equations involving decimals is inconvenient, the paper-and-pencil method is similar to the previous examples.

3. Why is modelling equations involving decimal numbers inconvenient?

Check your answer by turning to the Appendix.

Example 3

Solve $2n - 1.5n = 20$.

Solution

$2n - 1.5n = 20$ ← Combine like terms.

$0.5n = 20$

$\frac{0.5n}{0.5} = \frac{20}{0.5}$ ← Divide both sides by 0.5.

$n = 40$

Verification

LS	RS
$2n - 1.5n$	20
$= 2(40) - 1.5(40)$	
$= 80 - 60$	
$= 20$	

LS = RS

4. Solve and verify the following equations using paper and pencil.

a. $2x + 4x = 12$

b. $6y - 2y = -8$

c. $a - 3a = 6$

d. $-3y + 5y = 15$

e. $5a - 0.5a = 45$

f. $3.9a - 3.2a = 7$

5. Copy the boxes with the numbers in them into your notebook. Then solve the following equations. Place the letter that goes with each equation in the box with the correct solution to unscramble the message.

(A) $8y - 5y = 30$

(H) $4m + 3m = 84$

(L) $4a + 2a + 3a = 18$

(O) $8x + 2x - 2 = 28$

(V) $3x + 5x - 2x = 36$

(I) $12y - 11y = 8 - 3$

(T) $9b - 4b + 2b = 77$

(M) $10a - 5a - 7 + 5 = 33$

(E) $10c + 4c - 6c = 64$

5	2	3	6	8	7	10	11	12

Check your answers by turning to the Appendix.

Using Equations to Solve Problems

Do you recall writing equations to represent problem situations in previous mathematics courses? Now you will analyse some problems that can be represented by equations of the type you have just practised solving.

You will write the equation to represent the problem situation; then you will solve the equation and use the solution to answer the question in the problem.

Note: The solution to the equation will not always be the complete answer to the problem.

Example 1

The second of two numbers is seven times the first. Their sum is thirty-two. Find the numbers.

Solution

Write statements for the unknowns. Use only one variable.

Let n be the first number. Thus $7n$ is the second number.

"The second of two numbers is seven times the first."

Write an equation.

$$n + 7n = 32$$

"Their sum is thirty-two."

Solve the equation.

$$n + 7n = 32$$
$$8n = 32$$
$$\frac{8n}{8} = \frac{32}{8}$$
$$n = 4$$

The first number is 4.

Evaluate the second number.

$$7n = 7(4)$$
$$= 28$$

The second number is 28.

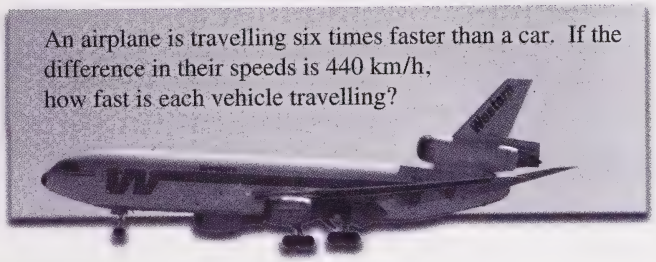
Verification

Reread the problem to make sure your answer is true.

Is 28 equal to 7 times 4? Yes.
Is the sum of 4 and 28 equal to 32? Yes.

Therefore, the numbers are 4 and 28.

Example 2


An airplane is travelling six times faster than a car. If the difference in their speeds is 440 km/h, how fast is each vehicle travelling?

Solution

Write statements for the unknowns. Use only one variable.

Let x be the speed of the car. Thus, $6x$ is the speed of the airplane.

The airplane travels six times faster.

Write an equation.

$$6x - x = 440$$

The difference in speed is 440 km/h.

Collect like terms and solve the equation.

$$6x - x = 440$$
$$5x = 440$$
$$\frac{5x}{5} = \frac{440}{5}$$
$$x = 88$$

The speed of the car is 88 km/h.

Now find the speed of the airplane.

$$6x = 6(88)$$
$$= 528$$

The speed of the airplane is 528 km/h.

Verification

Reread the problem to make sure your answer is true.

Is $528 - 88 = 440$? Yes.

Therefore, the car is travelling 88 km/h, and the airplane is travelling 528 km/h.

Solve the following problems using equations. Verify your answers.

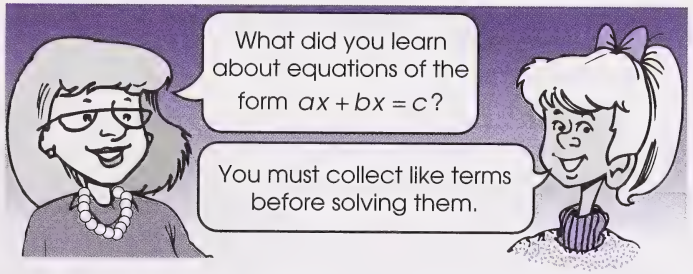
6. Two numbers differ by 18. The sum of the two numbers is 100. Find the numbers.

7. A basketball player had 70 free throws in a span of ten games. The number of missed free throws is six more than the number of free throws made. How many free throws did the player make?

8. Ramona, Jason, and Alexi sold pizzas to raise money for a class trip. They sold 91 pizzas in total. Ramona sold four times as many pizzas as Jason, and Alexi sold five fewer pizzas than Jason. How many pizzas did each student sell?

9. Marcia and Henry are working for an oil service company. A pipe with a length of 127 m is to be cut into three pieces. One piece is 17 m longer than the shortest piece. The third piece is three times longer than the shortest piece. How long are the three pieces of pipe?

Check your answers by turning to the Appendix.



Removing Parentheses

Some equations require you to remove parentheses before you can solve them.

View the video *Equations—Solving with More than One Step* from the *Math Moves* series, ACCESS. Watch the part entitled "Solving Equations: $a(x+b)=c$." Use the cut-out scale, counters, and cylinders from the Appendix to help you do the video assignment. This video is available from the Learning Resources Distributing Centre.

Check your answers to the video assignment by viewing the video.

Now that you have followed the solution of some equations involving parentheses on the video, compare these solutions to the following print examples.

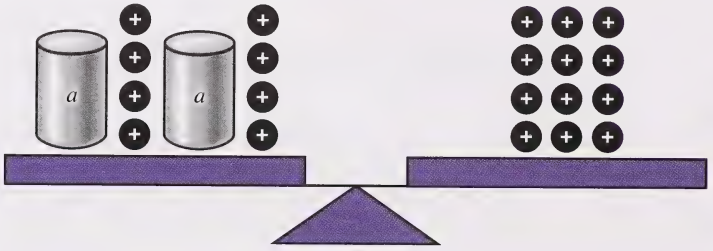
Study the examples and look for a pattern.

Example 1

Solve the equation $2(a+4)=12$.

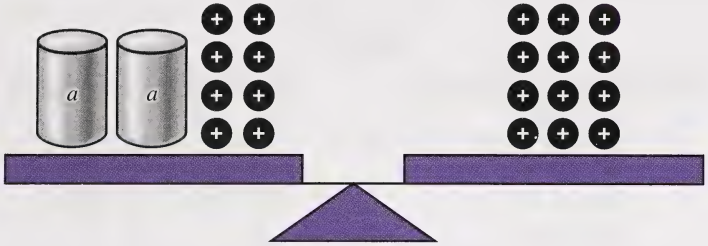
Solution

Model the equation.

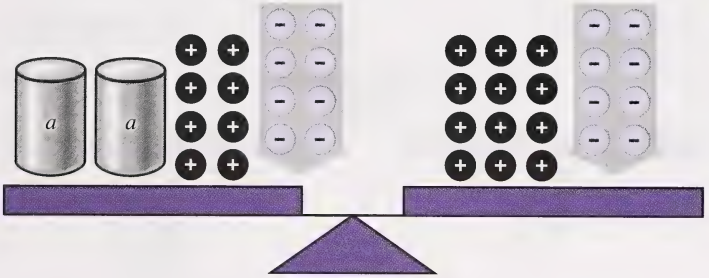


$2(a+4)$ means two groups of $a+4$.

Combine the groups. The model shows that $2(a+4)=2a+8$.



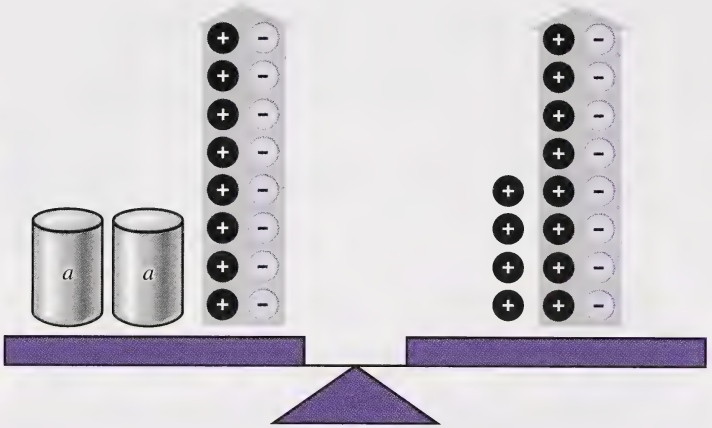
The additive inverse of 8 is -8. So, add -8 to both sides of the equation.



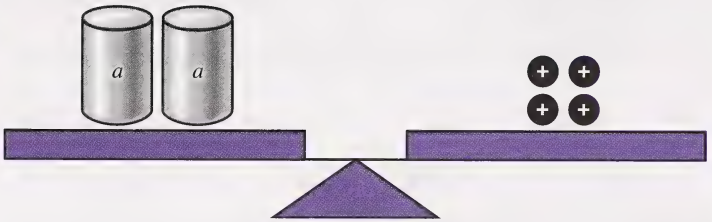
Why was -8 added to each side of the equation?

-8 was added to both sides in order to have only the variable on the left side.

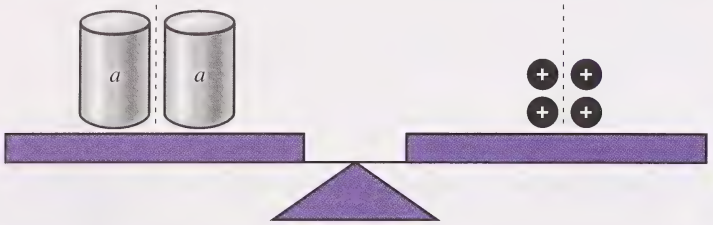
Simplify the equation by removing the zeros. This will not change the balance.



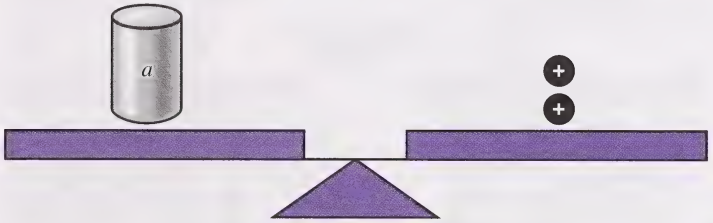
The result is as follows:



In order to isolate the variable, divide each side into two groups.



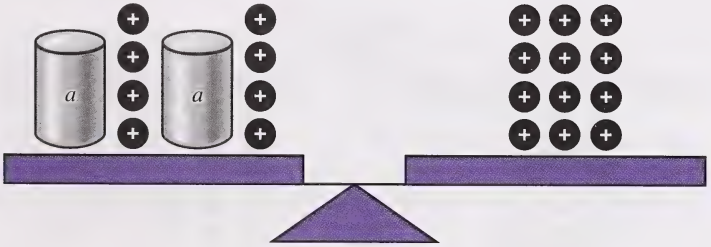
To solve the equation, examine only one group from each side.



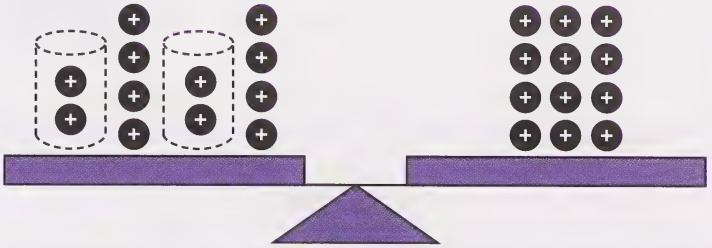
The solution is $a = 2$.

Verification

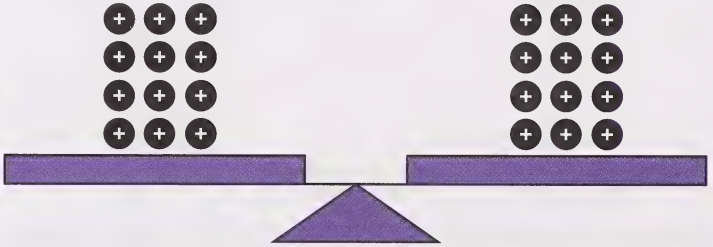
To verify the solution, model the equation.



Replace a with 2.



The result is as follows:



Both sides have a value of 12. The scale is balanced.

Therefore, the solution is $a = 2$.

You will now see how the equation in Example 1 is solved using paper and pencil.

Example 2

Solve the equation $2(a + 4) = 12$.

Solution

Write the equation.

$$2(a+4)=12$$

To remove the parentheses on the left side of the equation, multiply.

$$2a+8=12$$

To isolate a, subtract 8 from both sides.

$$2a+8-8=12-8$$
$$2a=4$$

To isolate a, divide both sides by 2.

$$\frac{2a}{2}=\frac{4}{2}$$
$$a=2$$

Verification

LS	RS
$2(a+4)$	12
$=2(2+4)$	
$=2(6)$	
$=12$	

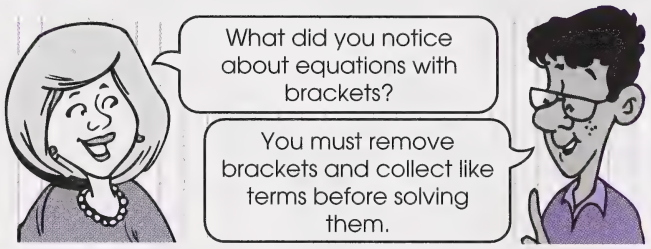
LS = RS

The solution is $a=2$.

10. a. How is the equation in Example 2 different from equations of the form $ax+b=c$?

b. When the brackets in the equation in Example 2 are removed and the like terms are collected, is the equation like those of the form $ax+b=c$?

Check your answers by turning to the Appendix.



11. Solve and verify the following equations using paper and pencil.

a. $3(y-2)=6$

b. $2(x+1)=-6$

c. $6(4+w)=18$

d. $3(7-4x)=29$

Check your answers by turning to the Appendix.

You can also use equations of this type to solve problems. The method is similar to that for the previous type of equation. Study the following example.

Example 3

The length of a rectangular swimming pool is 16 m more than its width. If the perimeter of the pool is 104 m, find the dimensions of the pool.

Solution

Let w be the width of the pool. Thus, $w+16$ is the length of the pool.

The equation for the perimeter is as follows:

$$2(w)+2(w+16)=104$$

Now, multiply to remove the parentheses.

$$\begin{aligned} 2w+2w+32&=104\\ 4w+32&=104\\ 4w+32-32&=104-32\\ 4w&=72\\ \frac{4w}{4}&=\frac{72}{4}\\ w&=18 \end{aligned}$$

Now find the length of the pool.

$$\begin{aligned} w+16&=18+16\\ &=34 \end{aligned}$$

Verification

Reread the problem.

Is the sum of two times the width (2×18 m) and two times the length (2×34 m) equal to the perimeter (104 m)? Yes.

Therefore, the width is 18 m, and the length is 34 m.

Now solve the following problems involving equations with brackets.

12. If you double the difference between a number and 7, the result is 25. Find the number.

13.

Amir wanted to go to a rock concert, but he didn't have enough money to buy a ticket. He had saved some money from his allowance over the past two weeks, and his father had given him $5 for helping to clean the garage; but Amir still needed more. Amir was able to triple his money by doing some odd jobs for his neighbour on the weekend. This left him with a total amount of $27, enough money to buy one ticket. How much had Amir saved from his allowance? How much did he make doing the odd jobs for his neighbour?

14. A 200 m length of chain is to be cut into four pieces. Two pieces have one length while the other two pieces have another length. The longer pieces are three times the length of the shorter pieces. How long is each piece of chain?

Check your answers by turning to the Appendix.

Sometimes it is helpful to be able to write a problem given an equation. It may help you to better understand how a problem and its equation are related.

Example 4

Write a problem represented by the equation $c + c + 3 + 3c = 104$, where c represents the cost of CDs purchased.

Solution

Colin, Jeena, and Boris bought compact disks at a music store. Jeena spent $3 more than Colin, and Boris spent three times as much as Colin. The total cost of the CDs was $104. How much did each person spend on CDs?

15. Write a problem represented by the equation $2p + p + 4p = 168$, where p represents the price of a shirt.

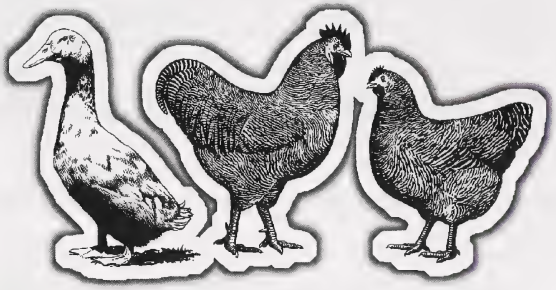
Check your answer by turning to the Appendix.

Now Try This

Now you are ready for an excursion.

Use one of the problem-solving strategies to solve the following problem.

16. Three chickens and one duck sold for as much as two geese. One chicken, two ducks, and three geese were sold together for $25. What was the price of each bird in an exact whole number of dollars?



Check your answer by turning to the Appendix.

Using Computer Spreadsheets to Solve Problems

A problem-solving strategy that was discussed in the Appendix of Module 1 is using a computer spreadsheet program. A computer spreadsheet program can be used in place of equations to solve problems. The following examples illustrate problem-solving using *ClarisWorks*™ (a spreadsheet program) for a Macintosh computer.
Note: You may use any spreadsheet program.

Example 1

One number is twelve times another number. The difference between the two numbers is eighty-eight. Find a pair of numbers that satisfies these conditions. How many other pairs of numbers satisfy these conditions?

Solution

Step 1: Set up an empty spreadsheet. Type First Number in cell A2, Second Number in cell A3, Sum in cell B1, and Product in cell C1.

Step 2: Let one number be placed in cell B2. Leave this cell blank until you have entered all the formulas.

Step 3: The other number can be placed in cell B3 and is given by the formula = B2 + 88.

Step 4: In column C set up a check for the product. Let cell C2 be the same as cell B2. Cell C2 will have the formula =B2.

Step 5: Cell C3 has the formula =12*C2.

Step 6: Try various numbers in cell B2 until cell C2 and cell C3 are the same.

Your spreadsheet, with the formulas entered, should look similar to the following:

	A	B	C
1		Sum	Product
2	First Number		=B2
3	Second Number	=B2+88	=12*C2
4			

Your spreadsheet, after you have solved the problem, should look similar to the following:

	A	B	C
1		Sum	Product
2	First Number	8	8
3	Second Number	96	96
4			

A pair of numbers that satisfies the conditions is 8 and 96. There are no other pairs of numbers that satisfy the conditions.

Example 2

The length of a rectangular swimming pool is 22.5 m greater than its width. If the perimeter of the pool is 131 m, what are the dimensions of the pool?

Solution

The following spreadsheet shows the information and formulas you need to enter in order to solve this particular problem.

	A	B
1	Dimensions of a Swimming Pool	
2		
3	Width	
4	Length	=B3+22.5
5	Perimeter	=2*(B3+B4)

Formulas have been entered for the length and perimeter.

Enter different values for the width (cell B3) until the spreadsheet calculates the value of 131 for the perimeter (cell B5).

Your spreadsheet, when you have solved the problem, should look as follows:

	A	B
1	Dimensions of a Swimming Pool	
2		
3	Width	21.5
4	Length	44
5	Perimeter	131

The dimensions of the pool are 21.5 m by 44 m.

Example 3

Jamie is saving nickels and dimes. He has 84 coins with a total value of $6.70. How many dimes and nickels does he have?

Solution

The following spreadsheet shows the information and formulas you need to enter in order to solve this particular problem.

	A	B	C
1	Nickels and Dimes		
2			
3		Number of Coins	Value
4	Nickels		=0.05*B4
5	Dimes	=84–B4	=0.10*B5
6	Total	=B4+B5	=C4+C5

Formulas have been entered for the number of dimes, the total number of coins, the dollar value of each type of coin, and the total dollar value.

Enter different values for the number of nickels until the total dollar value equals $6.70.

Your spreadsheet, when you have solved the problem, should look as follows:

	A	B	C
1	Nickels and Dimes		
2			
3		Number of Coins	Value
4	Nickels	34	$1.70
5	Dimes	50	$5.00
6	Total	84	$6.70

Jamie has 34 nickels and 50 dimes.

Now, solve the following problems using a spreadsheet program.

17. In a class of forty-three students, there are seven more girls than boys. How many boys and how many girls are there?

18. A change machine contains $287 in loonies and two-dollar coins. If there are 181 coins in all, how many loonies and how many two-dollar coins are there?

19. Jodie has 290 cm of picture framing material. If the length of the picture is 25 cm more than the width, what are the dimensions of the largest picture she can frame?

Check your answers by turning to the Appendix.

Spreadsheets can be used for many problems involving interest, age, amount of sales, and so on. For more information on using spreadsheets, do a word search for spreadsheets; then look for a basic spreadsheet tutorial. One such tutorial is available at the following URL.

http://forum.swarthmore.edu/sum95/math_and/spreadsheets/intro.html

This tutorial uses *ClarisWorks*™ for Macintosh.

Another tutorial is available at the following URL.

http://www1.omi.tulane.edu/classware/ic3/ss/sstop.html

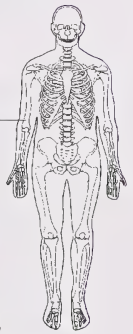
The discussion in this tutorial is suited to *ClarisWorks*™ as well as *Excel*® in both Macintosh and *Windows*™ versions.

Using Formulas

Some equations are used over and over again. These equations are often called **formulas**. Formulas are general rules that use letters to represent various values in each particular situation.

Example 1

Using the following formula, scientists can estimate the height of a female when the length of a section of bone, called the humerus, is known.



$$h = 2.75\ell + 71.4$$

In this formula, h is the height (in cm) of the individual and ℓ is the length (in cm) of the humerus.

If the length of the humerus of a certain female is 30 cm, then how tall is she? Round your answer to the nearest centimetre.

Solution

If $\ell = 30$, then $h = 2.75\ell + 71.4$

$$= 2.75(30) + 71.4$$
$$= 82.5 + 71.4$$
$$= 153.9$$

The female is about 154 cm tall.

Example 2

When a new drug is developed and the correct dosage for adults is determined, it is important to find the corresponding dosage for children. The following formula is used to determine the correct dosage for children (c) when the child's mass (m) and the adult dosage (a) is known.

$$c = \frac{m}{68} \times a$$

If a child's mass is 16 kg and the adult dosage is 15 mL, then what is the correct dosage for the child? Round your answer to the nearest tenth.

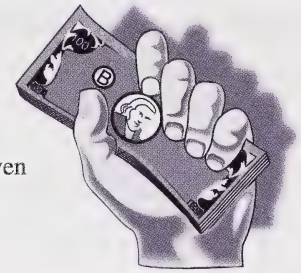
Solution

If $m = 16$ and $a = 15$, then $c = \frac{m}{68} \times a$

$$= \frac{16}{68} \times 15$$
$$= 3.529\,411\,765$$

The correct child's dosage is 3.5 mL, rounded to the nearest tenth.

20. When money is borrowed, there is usually a charge. This charge is called the interest (I), and the amount borrowed is called the principal (P). The amount of money paid to the lender (A) is given by the following formula.



$$A = P + I$$

a. If the principal is \$1000 and the interest is \$90, then what is the amount paid to the lender?

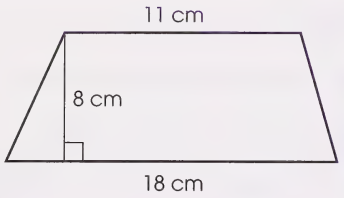
b. If the amount paid to the lender is \$1620 and the interest is \$120, what is the principal?

21. Sound travels at about 343 m/s at 20°C. The formula $d = 343t$ can be used to find the distance d (in metres) that sound travels in time t (in seconds) at 20°C.

a. There is a 6-s interval between the time Fatima sees lightning and hears thunder. Determine the distance Fatima is from the storm.

b. Randall is 5 km from a storm. What will be the time interval (to the nearest second) between the time he sees lightning and hears thunder? **Hint:** 1 km = 1000 m

22. Jasmin wants to find the area of the trapezoid shown. She uses the formula $A = \frac{1}{2}(b_1 + b_2) \times h$. Calculate the area for her.



23. Yin Yu wants to call her friend in Hong Kong. If she lives in Vancouver and calls between 18:00 and 06:00, the cost is $2.50 for the first three minutes and $1.25 for each additional minute.

a. Write the formula that can be used to represent this.

b. Calculate the cost of talking for 10 min.

c. Calculate the number of minutes Yin can talk for $20.

If you have access to a computer spreadsheet program, then use the spreadsheet to check your answers to questions 23.b. and 23.c.

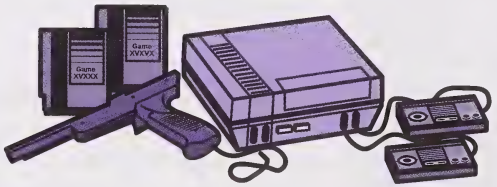
24. Find the puzzle "How Did the Doe Win the Big Animal Race?" in the Appendix. Either photocopy the puzzle or pull out the page; then complete the puzzle.

Check your answers by turning to the Appendix.

Now Try This

Use one of the problem-solving strategies to solve this problem.

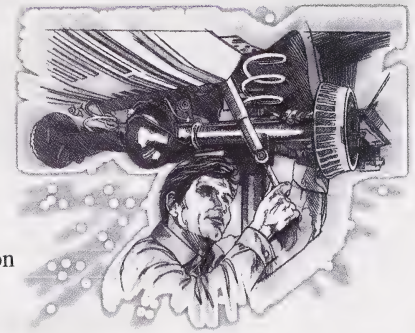
25. Tony and Yolanda both want to buy the same video game. Tony has $29 and can save $3.20 per day. Yolanda has $20 and can save $3.50 per day. If the cost of the game is $69, who will be able to buy the game first?



Check your answer by turning to the Appendix.

In this activity you solved equations that have the variables on one side. In some of the equations you simply collected like terms; in others, you needed to remove brackets before collecting like terms.

Activity 2: Solving Equations with Variables on Both Sides



Anton works at a local fast-food outlet. His father, Mr. Mironov, works as a licensed mechanic where he makes three times more per hour than his son. When Anton was lamenting about the discrepancy, his father told him not to worry; as soon as Anton's wage was increased by $11.60 per hour, they would be "equals." Find their hourly wages.

Solving a problem such as this requires an equation with variables on both sides of the equals sign. To solve these equations, you will have to separate the variables and the constants so that the variables are on one side of the equation and the constants are on the other side.

View the video *Equations—Solving with More than One Step* from the *Math Moves* series, ACCESS. Watch the part entitled "Solving Equations: $ax + b = cx$." Use the cut-out scale, counters, and cylinders from the Appendix to help you do the video assignment. This video is available from the Learning Resources Distributing Centre.

Check your answers to the video assignment by viewing the video.

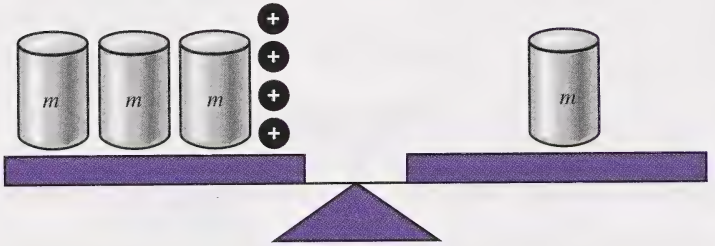
Study the following example and look for a pattern.

Example 1

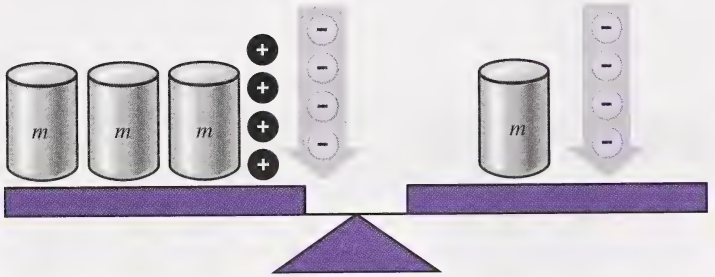
Solve the equation $3m + 4 = m$.

Solution

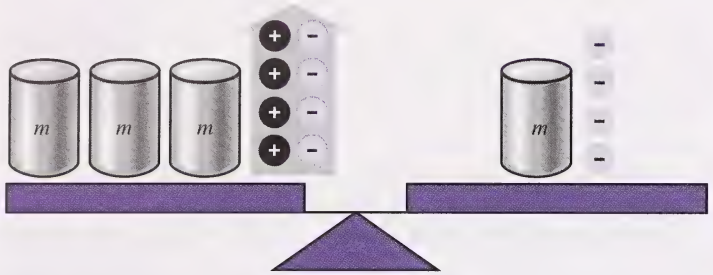
Model the equation.



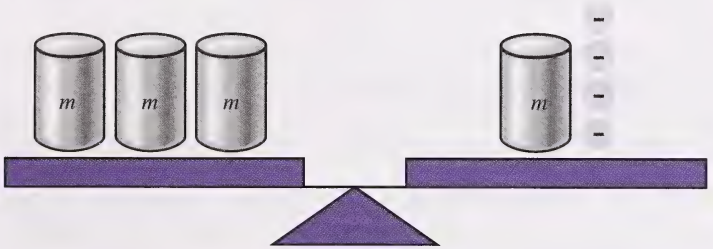
To isolate $3m$ on the left side, add -4 to both sides. -4 is the additive inverse of $+4$.



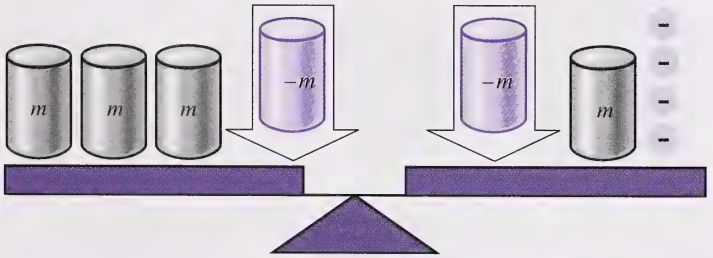
Simplify by removing the zeros.



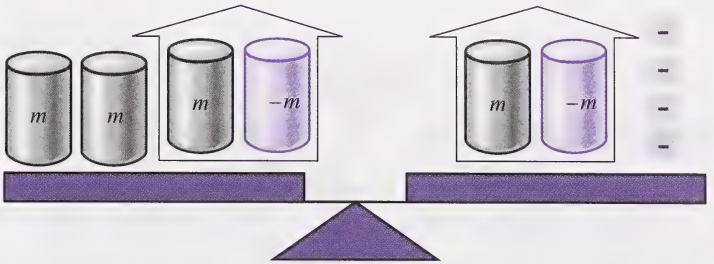
The result is as follows:



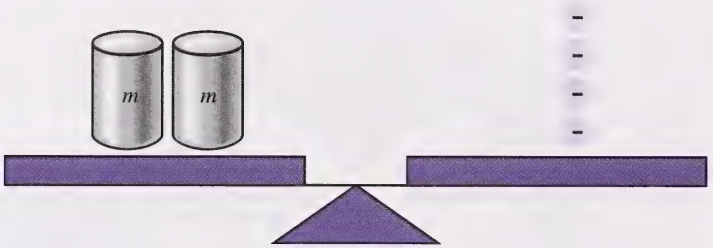
To isolate -4 on the right side, add $-m$ to both sides. $-m$ is the inverse of m.



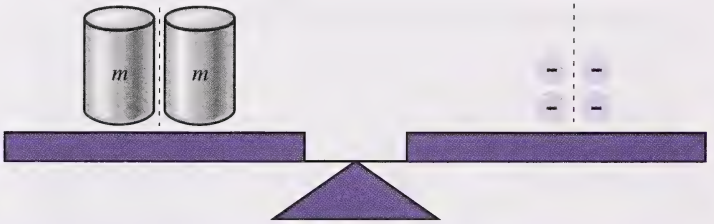
Remove the zeros.



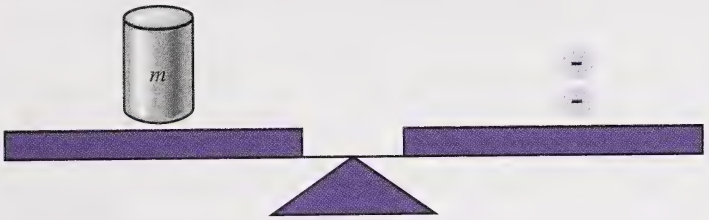
The result is as follows:



To isolate m, divide each side into two groups.



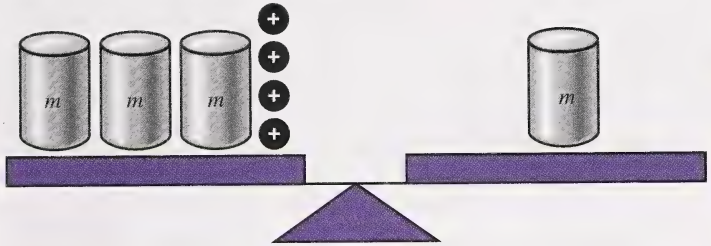
To solve the equation, examine only one group from each side.



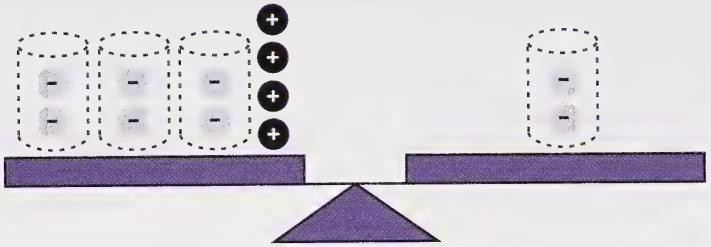
The solution is $m = -2$.

Verification

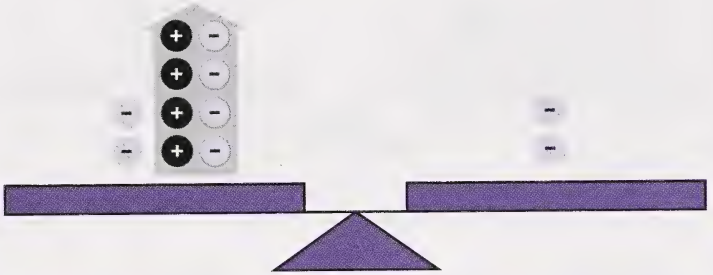
To verify the solution, model the equation.



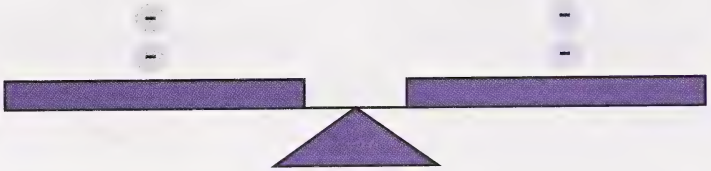
Replace m with -2.



Remove the zeros.



The result is as follows:



The value of each side of the equation is -2. The scale is balanced.

Therefore, the solution is $m = -2$.

1. The example you just examined models the solution of $3m+4=m$. There are variables on both sides of the equation.

   a. Why was -4 added to both sides of the equation?

   b. Why was $-m$ added to both sides of the equation?

   c. Why were both sides of the equation divided by 2?

Check your answers by turning to the Appendix.

Now, you will see how to solve Example 1 using paper and pencil.

Example 2

Solve the equation $3m+4=m$.

Solution

Write the equation.

$$3m+4=m$$

To isolate $3m$ on the left side of the equation, subtract 4 from both sides and simplify.

$$3m+4-4=m-4$$
$$3m=m-4$$

To isolate -4 on the right side of the equation, subtract m from both sides; then simplify.

$$3m-m=m-4-m$$
$$2m=-4$$

To isolate m, divide both sides by 2.

$$\frac{2m}{2}=\frac{-4}{2}$$
$$m=-2$$

Verification

LS	RS
$3m+4$	m
$=3(-2)+4$	$=-2$
$=-6+4$	
$=-2$	

LS = RS

The left side is equal to the right side; thus, the solution is $m=-2$.

2. Solve the following equations using paper and pencil.

a. $5n - 8 = 3n$

b. $10 - d = -6d$

c. $4n + 12 = -2n$

d. $3n + 48 = 5n$

e. $3.2a - 14 = 1.2a$

f. $1.6y = 2.8y - 6$

Check your answers by turning to the Appendix.

Sometimes there is a variable and a constant on each side of the equation. View the video and study the example to discover what additional steps are required.

View the video *Equations—Solving with More than One Step* from the *Math Moves* series, ACCESS. Watch the part entitled "Solving Equations: $ax + b = cx + d$." Use the cut-out scale, counters, and cylinders from the Appendix to help you do the video assignment. This video is available from the Learning Resources Distributing Centre.

Check your answers to the video assignment by viewing the video.

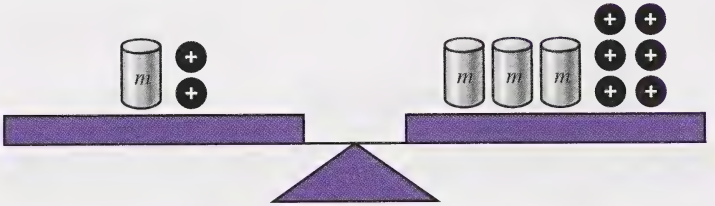
Now follow an example in which an equation with variables and constants on both sides is solved by modelling.

Example 3

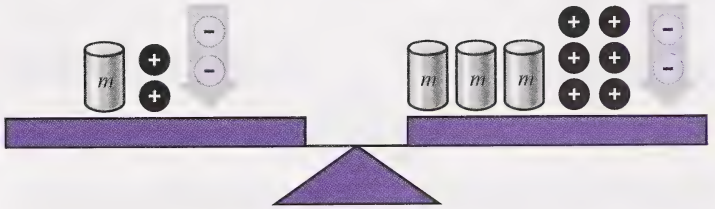
Solve the equation $m + 2 = 3m + 6$.

Solution

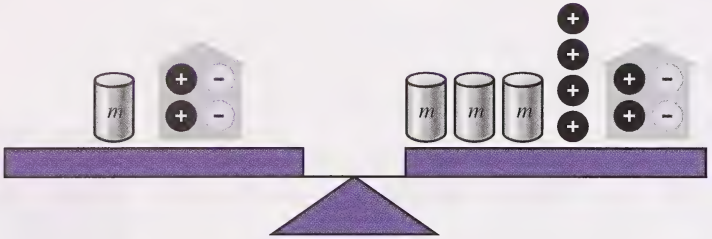
Model the equation.



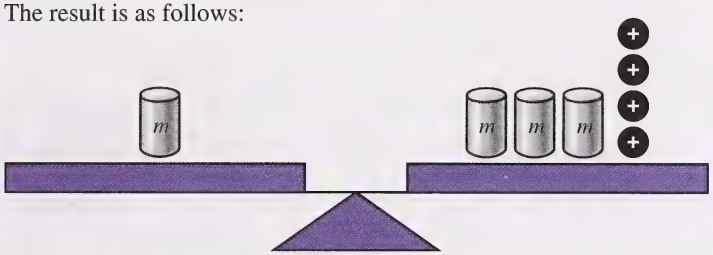
To isolate m on the left side of the equation, add -2 to both sides.



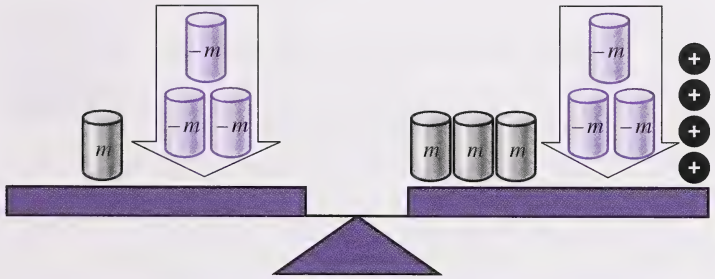
Remove the zeros.



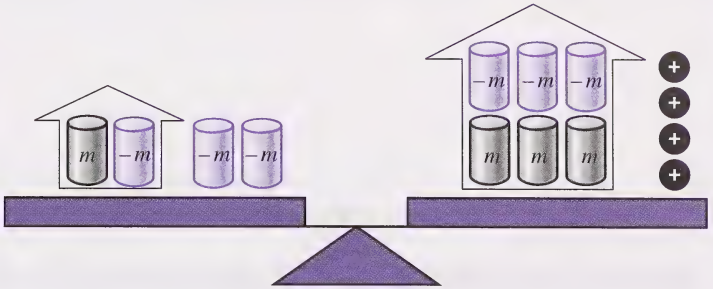
The result is as follows:



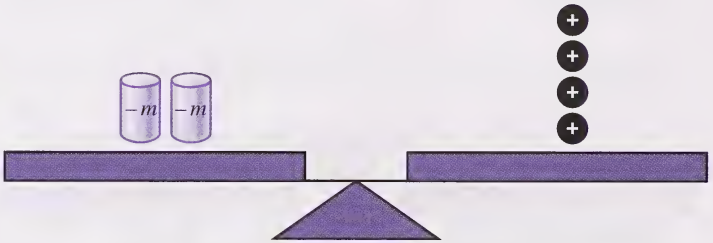
To isolate $+4$ on the right side of the equation, add $-3m$ to both sides.



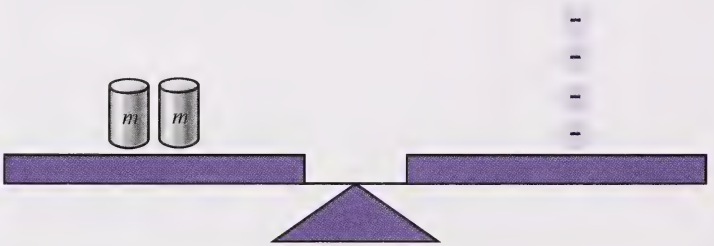
Remove the zeros.



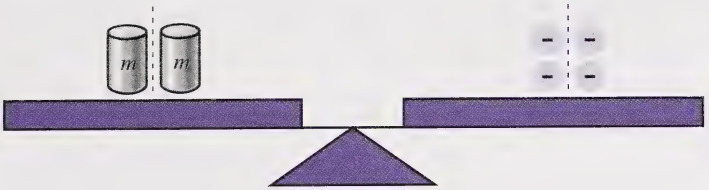
The result is as follows:



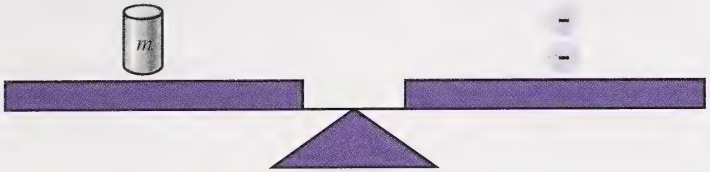
Replace each side with its inverse.



Divide each side into two groups.



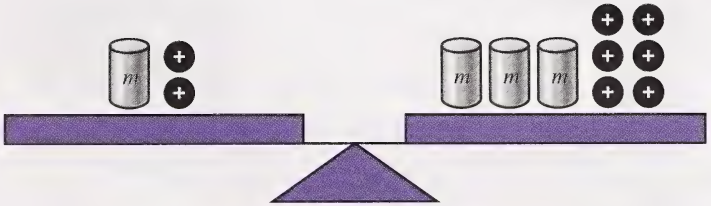
To solve the equation, examine only one group from each side.



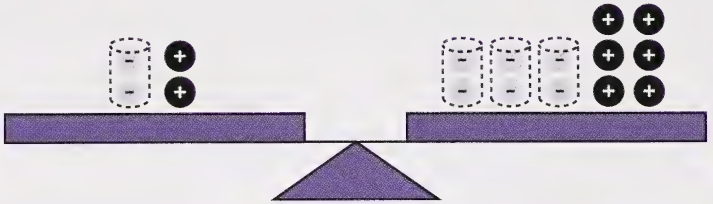
So, the solution is $m = -2$.

Verification

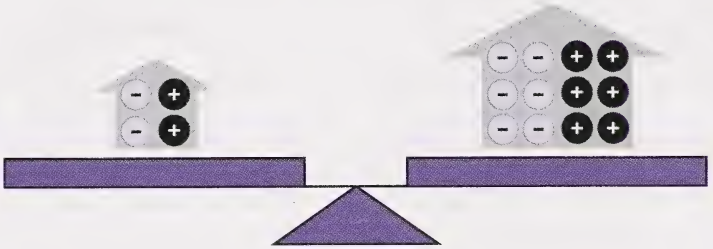
Model the equation.



Replace m with -2.



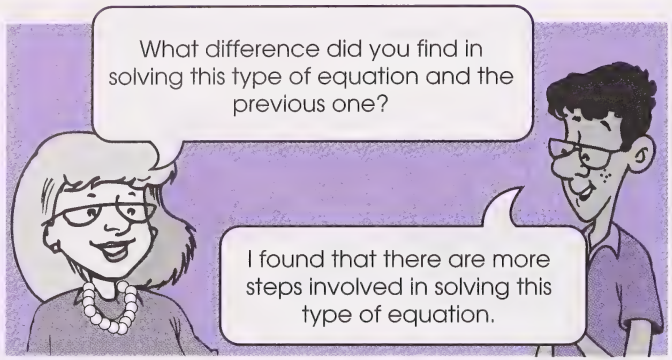
Remove the zeros.



The result is as follows:

The value of each side of the equation is zero. The scale is balanced.

Therefore, the solution is $m = -2$.


What difference did you find in solving this type of equation and the previous one?
I found that there are more steps involved in solving this type of equation.

Now you will see how to solve Example 3 using paper and pencil.

Example 4

Solve the equation $m+2=3m+6$.

Solution

Write the equation.

$$m+2=3m+6$$

To isolate the m on the left side, subtract 2 from both sides.

$$m+2-2=3m+6-2$$
$$m=3m+4$$

To isolate 4 on the right side, subtract $3m$ from both sides.

$$m-3m=3m+4-3m$$
$$-2m=4$$

To solve for m, divide both sides by -2.

$$\frac{-2m}{-2}=\frac{4}{-2}$$
$$m=-2$$

Verification

LS	RS
$m+2$	$3m+6$
$=-2+2$	$=3(-2)+6$
$=0$	$=0$

LS = RS

Thus, the solution is $m=-2$.

Now try an example that involves decimal numbers.

Example 5

Solve the equation $0.4q + 8.3 = 0.2q - 0.7$.

Solution

$$0.4q + 8.3 = 0.2q - 0.7$$

$0.4q + 8.3 - 8.3 = 0.2q - 0.7 - 8.3$ ← Subtract 8.3 from both sides.

$$0.4q = 0.2q - 9.0$$

$0.4q - 0.2q = 0.2q - 9.0 - 0.2q$ ← Subtract $0.2q$ from both sides.

$$0.2q = -9.0$$

$\frac{0.2q}{0.2} = \frac{-9.0}{0.2}$ ← Divide both sides by 0.2.

$$q = -45$$

Verification

LS	RS
$0.4q + 8.3$	$0.2q - 0.7$
$= 0.4(-45) + 8.3$	$= 0.2(-45) - 0.7$
$= -18 + 8.3$	$= -9 - 0.7$
$= -9.7$	$= -9.7$

LS = RS

Therefore, the solution is $q = -45$.

3. Solve the following equations using paper and pencil.

a. $3x + 9 = 2x - 5$ **b.** $4 - 8a = 2a - 66$

c. $3 + 2a = -5a - 11$ **d.** $8n - 2 = 6n + 8$

e. $2.6c + 7.6 = 1.2c - 8.3$ (Round to one decimal place.)

f. $1.1 + 3.2s = 5s + 4.3$ (Round to one decimal place.)

4. Explain how you would model the solution of the equation $3x - 2 = x$.

5. Solve the problem posed at the start of this activity.

6. The equations $P_1 = 5.90 + 0.90t$ and $P_2 = 7.25 + 0.45t$ represent the cost of a small pizza from two different pizza shops, where P is the price of a pizza and t is the number of toppings. Use the equations to determine the number of toppings for which the cost is equal.

7. Find the puzzle "Books Never Written" in the Appendix. Either photocopy the puzzle or pull out the page; then complete the puzzle. Use equations to help you solve each problem.

8. Write a problem to represent the equation $n + 17.40 = 4n$, where n is Lisa's hourly wage and $4n$ is her supervisor's hourly wage.

Check your answers by turning to the Appendix.

Equations with Variables on Both Sides that Involve Brackets

Sometimes you will see equations that look quite complicated because they have variables and constants on each side of the equation and they also contain brackets. These equations are not that different or complicated. They are just a combination of types of equations that you have already solved.

Example 1

Solve the equation $4(2x-3)=6(x-1)$.

Solution

First write the equation.

$$4(2x-3)=6(x-1)$$

Do the indicated multiplication on both sides of the equation to remove the parentheses.

$$8x-12=6x-6$$

Now it looks like the type just completed.

To isolate $8x$ on the left side of the equation, add 12 to both sides.

$$8x-12+12=6x-6+12$$
$$8x=6x+6$$

To isolate $+6$ on the right side of the equation, subtract $6x$ from both sides.

$$8x-6x=6x+6-6x$$
$$2x=6$$

To isolate x, divide both sides by 2.

$$\frac{2x}{2}=\frac{6}{2}$$
$$x=3$$

Verification

LS	RS
$4(2x-3)$	$6(x-1)$
$=4[2(3)-3]$	$=6(3-1)$
$=4(6-3)$	$=6(2)$
$=4(3)$	$=12$
$=12$	

LS = RS

Therefore, the solution is $x=3$.

Example 2

Solve the equation $3(-2x+4)=-4(x-6)$.

Solution

$$3(-2x+4)=-4(x-6)$$ ← Remove the brackets first; multiply.

$$-6x+12=-4x+24$$

$$-6x+12-12=-4x+24-12$$ ← Subtract 12 from both sides.

$$-6x=-4x+12$$

$$-6x+4x=-4x+12+4x$$ ← Add $4x$ to both sides.

$$-2x=12$$

$$\frac{-2x}{-2}=\frac{12}{-2}$$ ← Divide both sides by -2.

$$x=-6$$

Verification

LS	RS
$3(-2x+4)$	$-4(x-6)$
$=3[-2(-6)+4]$	$=-4(-6-6)$
$=3(12+4)$	$=-4(-12)$
$=3(16)$	$=48$
$=48$	

LS = RS

Therefore, the solution is $x=-6$.

9. Solve the following equations using paper and pencil. Verify your answers.

a. $5(3x-8)=2(2x+13)$ **b.** $4(a+6)=8(a+4)$

c. $-4(2x-3)=2(-x+3)$ **d.** $0.6(2h+6)=1.4(h+2)$

10. Freda's Uncle Julius would not reveal his age. However, being a mathematics teacher, he admitted that in 9 years he would be twice as old as he was 20 years ago. How old is he?

11. The Steinsens are fencing an area in the backyard in which to keep their two dogs. They have 56 m of chain link fencing and would like the length of the enclosure to be 4 m more than twice the width. What are the dimensions of the enclosure?

Check your answers by turning to the Appendix.

As the equations become more complicated, it is important to develop different strategies to solve problems.

Using Tables as a Problem-Solving Strategy

Even though you are using an equation to represent the situation of a problem, sometimes it helps to draw a table or diagram to organize your thoughts and plans when attempting to solve the problem. A table can be used to show relationships, thus helping in setting up the equation.

Example 1

Mrs. Sanchez is nine times older than her daughter Carmen. In three years Mrs. Sanchez will only be five times older than her daughter. Find their present ages.

Solution

Use a table to organize the information.

Person	Present Age	Age in Three Years
Mrs. Sanchez	$9n$	$9n+3$
Carmen	n	$n+3$

Add three years to their present ages.

In three years Mrs. Sanchez will be five times Carmen's age. Use this information to help you write and solve the equation.

$$\begin{aligned} 9n+3 &= 5(n+3) \\ 9n+3 &= 5n+15 \\ 9n+3-3 &= 5n+15-3 \\ 9n &= 5n+12 \\ 9n-5n &= 5n+12-5n \\ 4n &= 12 \\ \frac{4n}{4} &= \frac{12}{4} \\ n &= 3 \end{aligned}$$

Now find Mrs. Sanchez's present age.

$$\begin{aligned} 9n &= 9(3) \\ &= 27 \end{aligned}$$

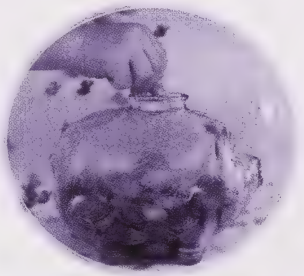
Looking Back

Reread the problem and be sure your answer is true.

In three years Mrs. Sanchez will be 30 years old and Carmen will be 6 years old. Is 30 equal to 5 times 6? Yes.

Therefore, Carmen is presently 3 years old, and Mrs. Sanchez is presently 27 years old.

Example 2



Ahmed and Danny were counting the money in their piggy banks one day. They discovered they had exactly the same amount of money. Ahmed had only dimes and nickels in his piggy bank and Danny had only quarters and pennies in his. Ahmed had two more than three times as many nickels as dimes. Danny had the same number of quarters as Ahmed had dimes and three more pennies than he had quarters. How much money did each boy have in his piggy bank?

Solution

Use a table to organize the information.

Person	Type of Coin	Number of Coins	Value of One Coin (¢)	Total Value
Ahmed	dime	d	10	$10d$
	nickel	$3d+2$	5	$5(3d+2)$
Danny	quarter	d	25	$25d$
	penny	$d+3$	1	$d+3$

Since the boys have the same amount of money, you can write the following equation.

$$10d+5(3d+2)=25d+(d+3)$$

Solve the equation and find the answer to the problem.

$$\begin{aligned}10d+5(3d+2)&=25d+(d+3)\\10d+15d+10&=25d+d+3\\25d+10&=26d+3\\25d+10-10&=26d+3-10\\25d&=26d-7\\25d-26d&=26d-7-26d\\-d&=-7\\d&=7\end{aligned}$$

Now find the amount of money each boy has in his piggy bank. Since they both have the same amount of money, you only need to determine the amount in one piggy bank. Determine the amount Ahmed has.

$$\begin{aligned}10d+5(3d+2)&=10(7)+5\left[3(7)+2\right]\\&=70+5(21+2)\\&=70+5(23)\\&=70+115\\&=185\end{aligned}$$

Verification

Check to make sure Danny has the same amount of money as Ahmed.

$$\begin{aligned}25d+(d+3)&=25(7)+(7+3)\\&=175+10\\&=185\end{aligned}$$

Therefore, Ahmed and Danny each have 185¢ (or $1.85) in their piggy banks.

Solve the following problems. Use equations to represent the situation in each problem and tables to help you organize the information.

12. Vladimir's age is double the sum of Enzo's age and 4. Two years ago he was three times Enzo's age. Find their present ages.

13. Mrs. Waslyk, Mrs. Jackson, and Mrs. Winkler were discussing investments on the drive to work. They discovered that they all had purchased stocks in the same company. Mrs. Jackson's initial investment was five times more than Mrs. Waslyk's. Mrs. Winkler's was three times the difference between Mrs. Waslyk's and $20. The difference between what Mrs. Jackson had invested and what Mrs. Winkler invested was $1000 more than what Mrs. Waslyk had invested.

a. What was each lady's initial investment?

b. If Mrs. Waslyk bought 400 shares initially, what was the cost per share?

c. If Mrs. Waslyk's investment is now worth $1300, how much has each share increased in price?

d. What is the percent increase in the value of Mrs. Waslyk's shares? Round your answer to the nearest percent.

14. A vending machine, when emptied by the attendant at the end of the week, contained the following distribution of coins: twice as many dimes as nickels, four times as many quarters as dimes, and ten more loonies than quarters. If the attendant collected $133, how many of each type of coin were in the machine?

15. Find the puzzle "What Did They Call the Bug that the Astronauts Brought Back from the Moon?" Either photocopy the puzzle or pull out the page; then complete the puzzle.

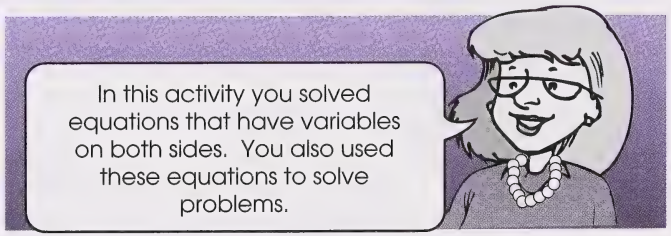
Check your answers by turning to the Appendix.

Now Try This

Use one of the problem-solving strategies to solve the following problem.

16. Elsie wants to get a photograph enlarged so that the length and width are twice those of the original. She knows the original cost of the print is $1.60 and is surprised when she is told the cost of the enlargement will be $6.40. Elsie assumed the cost would be twice the original or $3.20. As an employee of the photograph shop, how would you explain to Elsie in terms of the areas of the two photographs why the cost is $6.40?

Check your answer by turning to the Appendix.


In this activity you solved equations that have variables on both sides. You also used these equations to solve problems.

Activity 3: Solving Equations Containing Fractions

Mrs. Nettleton buys some fruit to make a salad. Grapes cost half as much as cherries, and strawberries cost a third as much as cherries. If she bought 1 kg of each kind of fruit and the total was $17.38, how much does it cost per kilogram for each kind of fruit?

To answer this question you are going to have to use an equation that involves fractions.

Up to this point you have solved equations in which the variable was multiplied by an integer, as shown in this example.

$$3a - 8 = -2a + 7$$

You know that $3a$ means $3 \times a$ and $-2a$ means $-2 \times a$.

1. What is the coefficient of $3a$ and $-2a$?

2. Now look at an equation involving fractions such as $\frac{n}{3} = \frac{7}{4}$.

a. What does $\frac{n}{3}$ mean?

b. What is the coefficient of $\frac{n}{3}$?

3. The coefficient of $\frac{4}{m}$ is 4. Explain why.

4. What is the coefficient of $\frac{n+4}{6}$? Why?

Check your answers by turning to the Appendix.

There are a number of different ways of solving equations containing fractions. Note each method carefully as you work through Examples 1 and 2.

Example 1

Helen is picking wild strawberries. She picks 1 container in 2 h. How many containers can she pick in 5 h?

Solution

Let p be the number of containers she can pick in 5 h.

Thus, the equation is $\frac{p}{5} = \frac{1}{2}$.

Solve $\frac{p}{5} = \frac{1}{2}$.

Method 1: Using the Multiplicative Inverse

Write the equation.

$$\frac{p}{5}=\frac{1}{2}$$

$\frac{p}{5}$ means $\frac{1}{5}p$.

To isolate the variable, multiply both sides by 5.

$$\frac{p}{\cancel{5}_{1}}\times\cancel{5}^{1}=\frac{1}{2}\times 5$$

$$p=\frac{5}{2}\text{ or }2.5$$

Method 2: Clearing the Denominators

Write the equation.

$$\frac{p}{5}=\frac{1}{2}$$

Multiply both sides of the equation by 10 (the product of the denominators 2 and 5 or the LCM). This is called **clearing the denominators**.

$$\frac{p}{5}\times 10=\frac{1}{2}\times 10$$

Simplify each side of the equation.

$$\frac{p}{\cancel{5}_{1}}\times\cancel{10}^{2}=\frac{1}{\cancel{2}_{1}}\times\cancel{10}^{5}$$

$$2p=5$$

To isolate the variable, divide both sides by 2.

$$\frac{2p}{2}=\frac{5}{2}$$

$$p=\frac{5}{2}\text{ or }2.5$$

Method 3: Using Cross Products

Write the equation.

$$\frac{p}{5}=\frac{1}{2}$$

Find the **cross products**. This is a shortcut to clearing the denominators.

$$\frac{p}{5}\quad\frac{1}{2}$$

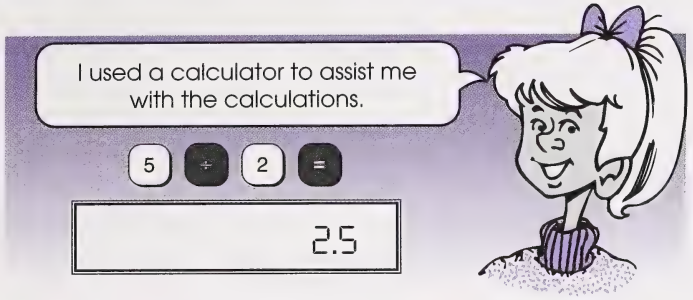
$$p\times 2=1\times 5$$

$$2p=5$$

To isolate the variable, divide both sides by 2.

$$\frac{2p}{2} = \frac{5}{2}$$

$$p = \frac{5}{2} \text{ or } 2.5$$



Verification

LS	RS
$\frac{p}{5}$	$\frac{1}{2}$
$= \frac{2.5}{5}$	$= 0.5$
$= 0.5$	

LS = RS

Therefore, Helen can pick 2.5 containers of strawberries in 5 h.

In some equations, the variable may be in the denominator.

Example 2

Solve the equation $\frac{3}{a} = \frac{5}{4}$.

Solution

Method 1: Using the Multiplicative Inverse

$$\frac{3}{a} = \frac{5}{4}$$ ← Write the equation.

$$\frac{3}{\cancel{a}_{1}} \times \overset{1}{\cancel{a}} = \frac{5}{4} \times a$$ ← Multiply both sides by a.

$$3 = \frac{5a}{4}$$

$$3 \times 4 = \frac{5a}{\cancel{4}_{1}} \times \overset{1}{\cancel{4}}$$ ← Multiply both sides by 4.

$$12 = 5a$$

$$5a = 12$$

$$\frac{5a}{5} = \frac{12}{5}$$ ← Divide both sides by 5.

$$a = \frac{12}{5} \text{ or } 2.4$$

Method 2: Clearing the Denominators

$$\frac{3}{a} = \frac{5}{4}$$ ← Write the equation.

$$\frac{3}{\cancel{a}_{1}} \times 4\cancel{a}^{1} = \frac{5}{\cancel{4}_{1}} \times \cancel{4}^{1}a$$ ← Multiply both sides by $4a$.

$$12 = 5a$$

$$5a = 12$$

$$\frac{5a}{5} = \frac{12}{5}$$ ← Divde both sides by 5.

$$a = \frac{12}{5} \text{ or } 2.4$$

Method 3: Using Cross Products

$$\frac{3}{a} = \frac{5}{4}$$ ← Write the equation.

$$\frac{3}{a} \times \frac{5}{4}$$ ← Find the cross products.

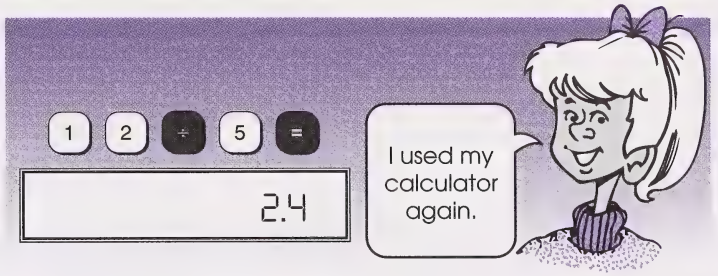
$$3 \times 4 = 5 \times a$$

$$12 = 5a$$

$$5a = 12$$

$$\frac{5a}{5} = \frac{12}{5}$$ ← Divde both sides by 5.

$$a = \frac{12}{5} \text{ or } 2.4$$



Verification

LS	RS
$\frac{3}{a}$	$\frac{5}{4}$
$= \frac{3}{2.4}$	
$= \frac{3 \times 10}{2.4 \times 10}$	
$= \frac{30}{24}$	
$= \frac{5}{4}$	

LS = RS

Therefore, the solution is $a = 2.4$.

Although Method 1 does yield a correct solution, you will find that Methods 2 and 3 are easier to use when the variable is in the denominator.

Answer the following questions to check your understanding of solving simple equations containing fractions. Use the method indicated in each question. Use your calculator as required.

5. What number should both sides of the equation be multiplied by in order to isolate the variable in each of the following?

a. $\frac{n}{15} = \frac{2}{3}$ **b.** $\frac{f}{9} = \frac{8}{3}$

c. $\frac{y}{5} = \frac{5}{6}$ **d.** $\frac{3}{4} = \frac{c}{20}$

6. Solve the equations in question 5 using the multiplicative inverse method. Verify your answers.

7. Solve each of the following equations by clearing the denominators. Verify your answers.

a. $\frac{n}{5} = \frac{3}{4}$ **b.** $\frac{2}{n} = \frac{5}{8}$ **c.** $\frac{x}{3} = \frac{4}{5}$ **d.** $\frac{3}{r} = \frac{5}{8}$

8. Solve the equations in question 7 by using cross products.

9. Explain, in words, how to solve the following equations using the method indicated.

a. Solve $\frac{r}{4} = \frac{3}{5}$ by clearing the denominators.

b. Solve $\frac{8}{11} = \frac{5}{y}$ by using cross products.

10. Indicate the keys you would press on a calculator in order to solve the equations in question 8.

11. Marvin can pick 32 containers of wild blueberries in 5 h.

a. How many containers can he pick in 8 h?

b. How long will it take him to pick 112 containers?

12. The number 24 is divided into two parts such that the ratio of the two parts is 5 to 7. If the larger part is 4 more than the smaller part, find the two parts.

13. Did you know that you can find the temperature by counting the chirps of a cricket using the following formula?

$$t = \frac{n}{8} + 5$$

In this formula, t is the temperature (in °C) and n is the number of chirps in one minute.

a. Find the temperature if Ruth counts 80 chirps in one minute.

b. How many times should a cricket chirp at a temperature of 25°C?

Use a scientific calculator to answer question 14.

14. The formula $T = t - \frac{h}{150}$ is used to find the temperature at a given altitude. T is the air temperature (in °C), h is altitude (in metres), and t is the ground temperature (in °C).

a. Calculate the temperature outside an aircraft at altitudes of 12 000 m and 4000 m if the ground temperature is 15°C. Round your answers to the nearest degree where necessary.

b. Calculate the temperature outside an aircraft at an altitude of 4000 m if the ground temperature is $-30°\text{C}$.

Check your answers by turning to the Appendix.

Rearranging Formulas

Throughout this module you have solved a number of equations involving formulas.

It is sometimes necessary or convenient to rearrange a formula to find the value of a certain variable—that is, to express the formula in terms of the missing variable. Often when you rearrange the formula the result is an equation in fraction form. The following examples illustrate two ways of finding the missing value. You should be familiar with both methods.

Example

The formula $d = vt$ is used to find the distance travelled when the average rate of speed and the time travelled are known. In this formula, d is the distance travelled (in km), v is the average rate of speed (in km/h), and t is the time travelled (in h).

If Mrs. Rhodes drove 400 km in 5 h, what was her average rate of speed?

Solution

Method 1

Solve for v, then substitute the known values.

$$d = vt$$

$$\frac{d}{t} = \frac{vt}{t} \quad \leftarrow \text{Divide both sides by } t \text{ to isolate } v.$$

$$\frac{d}{t} = v$$

$$\therefore\ v = \frac{d}{t}$$

If $d = 400$ and $t = 5$, then $v = \frac{d}{t}$

$$= \frac{400}{5}$$

$$= 80$$

Mrs. Rhodes travelled at an average rate of speed of 80 km/h.

Method 2

Substitute the known values, and solve for v.

If $d = 400$ and $t = 5$, then

$$d = vt$$
$$400 = v \times 5$$
$$5v = 400$$
$$\frac{5v}{5} = \frac{400}{5} \quad \leftarrow \text{Divide both sides by 5 to isolate } v.$$
$$v = 80$$

Mrs. Rhodes travelled at an average rate of speed of 80 km/h.

Do the following questions involving rearranging formulas.

15. Express each of the following formulas in terms of the variable shown in parentheses. Write the rearranged formula with the indicated variable on the left of the equal sign.

a. $C = \pi d \quad (d)$

b. $P = a + b + c \quad (a)$

c. $A = 2b + c \quad (c)$

d. $A = \frac{1}{2}ab \quad (b)$

e. $I = Prt \quad (P)$

f. $V = \ell wh \quad (\ell)$

g. $A = \frac{1}{2}(b_1 + b_2) \times h \quad (h)$

16. Find the value of the missing variable for each of the following situations.

a. Using the formula from question 15.a., find d if $C = 27.004$ cm.

b. Using the formula from question 15.b., find a if $P = 48.5$ cm, $b = 7.6$ cm, and $c = 19.4$ cm.

c. Using the formula from question 15.d., find b if $A = 63.6 \text{ cm}^2$ and $a = 10.6$ cm.

d. Using the formula from question 15.f., find ℓ if $V = 336 \text{ cm}^3$, $w = 8.4$ cm, and $h = 3.2$ cm.

e. Using the formula from question 15.g., find h if $A = 126 \text{ cm}^2$, $b_1 = 12$ cm, and $b_2 = 18$ cm.

17. Explain how $d = vt$ and $v = \frac{d}{t}$ are related.

18. The Langfield School hockey team is sponsoring their zones hockey tournament.

The following formula is used to find the cost of printing hockey tickets.

$$c = 50 + 0.1n$$

In this formula, c is the cost of printing (in $) and n is the number of tickets printed.

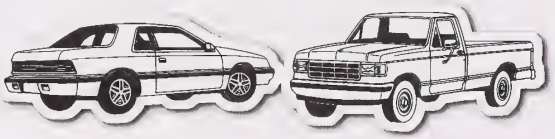
If the cost of printing the tickets was $350, then how many tickets were printed?

Check your answers by turning to the Appendix.

Now Try This

Use one of the problem-solving strategies to solve the following problem.

19. There are 150 vehicles parked in the storage lot of a local dealership. Of these vehicles, four-fifths are new; of these new vehicles, three-quarters are cars; of these cars, two-thirds are American made; and, of these American-made cars, one-half have air conditioning.



a. How many of the vehicles in the lot are new, American-made cars with air conditioning?

b. Find the answer to question 19.a. without finding the number of vehicles with the desired characteristics as you go down the list.

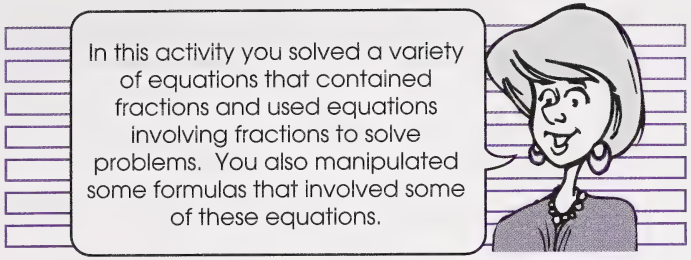
c. If there are 24 new, American-made cars with air conditioning in the lot and the same fractions apply, then how many new vehicles would there be?

Check your answers by turning to the Appendix.

If you have access to the Internet, you may wish to study some additional information involving solving single-variable equations. You can access a site for junior high mathematics by entering the following URL.

http://forum.swarthmore.edu/dr.math/drmath.middle.html

Click on topics like Word Problems or Equations.


In this activity you solved a variety of equations that contained fractions and used equations involving fractions to solve problems. You also manipulated some formulas that involved some of these equations.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

Using Problem-Solving Strategies

At the beginning of each module, you are given a list of various strategies that you can use to solve problems. (These strategies are explained, with the use of examples, in the Appendix of Module 1.) You have already encountered some situations where some of these strategies were used to solve problems in this and previous modules. You discovered and used the equations in this module. The problems that you solved using equations may also be solved using other methods.

Example 1

Yvonne has 20 more nickels than dimes in her piggy bank. If the total value of the nickels and dimes is $8.50, how many coins of each type does she have?

Solution

To help you understand the problem, simplify it. Here is a similar, but simpler, problem.

> Yvonne has 10 nickels and 3 dimes in her bank. What is the total value of the coins?
>
> Since it is easier to work with whole numbers than decimal numbers, express the value of the coins in cents. You may wish to use a table to organize the data.

Type of Coin	Number of Coins	Value of One Coin (¢)
Dimes	3	10
Nickels	10	5

$$(10 \times 5) + (3 \times 10) = 50 + 30$$
$$= 80$$

Yvonne has 80¢ (or \$0.80) in her piggy bank.

Solving the simpler problem gives you a better understanding of the original problem—the total value of the coins is the sum of the product of the number of nickels and the value of a nickel, and the product of the number of dimes and the value of a dime. Now return to the original problem in Example 1. There are several ways to solve the problem once you understand it.

Method 1: Guessing, Checking, and Revising

You can make a guess and test to see if the guess is correct.

Guess 1

Type of Coin	Number of Coins	Value (¢)
Dimes	10	100
Nickels	30	150
Total		250

The total of 250¢ (or \$2.50) is too low; thus, there are more nickels and dimes.

Guess 2

Type of Coin	Number of Coins	Value (¢)
Dimes	20	200
Nickels	40	200
Total		400

The total of 400¢ (or \$4.00) is still too low; thus, there are more nickels and dimes.

Guess 3

Type of Coin	Number of Coins	Value (¢)
Dimes	50	500
Nickels	70	350
Total		850

This total is correct.

Yvonne has 50 dimes and 70 nickels in her piggy bank.

Method 2: Using an Equation

Let the number of dimes be n.

The number of nickels, the value of the nickels, the number of dimes, and the value of the dimes must also be represented by algebraic expressions.

A table can be used to organize the information.

Type of Coin	Number of Coins	Value (¢)
Dimes	n	$10n$
Nickels	$n+20$	$5(n+20)$
Total		850

The value is 10 times the number of dimes.

The value is 5 times the number of nickels.

Yvonne has 20 more nickels than dimes.

This is given.

The total value of the nickels and dimes is \$8.50 (or 850¢).

Write an equation.

$$5(n+20)+10n=850$$

Solve the equation.

$$\begin{aligned} 5(n+20)+10n &= 850 \\ 5n+100+10n &= 850 \\ 15n+100 &= 850 \\ 15n+100-100 &= 850-100 \quad \text{← Subtract 100 from both sides.} \\ 15n &= 750 \\ \frac{15n}{15} &= \frac{750}{15} \quad \text{← Divide both sides by 15.} \\ n &= 50 \end{aligned}$$

Now find the number of nickels in Yvonne's piggy bank.

$$\begin{aligned} n+20 &= 50+20 \\ &= 70 \end{aligned}$$

Therefore, Yvonne has 50 dimes and 70 nickels in her piggy bank.

The following example will show you an alternative way of solving Example 1, under the heading **Using Tables as a Problem-Solving Strategy** in Activity 2 of this section.

Example 2

Mrs. Mahr is nine times the age of her daughter. In three years Mrs. Mahr will only be five times her daughter's age. What are their ages now?

Solution

To help you understand the problem, you may wish to simplify it. Here is a similar but simpler problem.

Mrs. Mahr is 33 years old and her daughter is 3 years old. How many times her daughter's age is Mrs. Mahr now? In three years, how many times her daughter's age will Mrs. Mahr be?

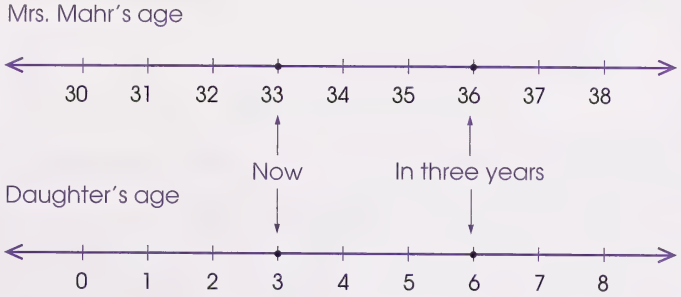
Mrs. Mahr is 11 times her daughter's age now.

$$33=11\times 3$$

In three years Mrs. Mahr will be 6 times her daughter's age.

$$(33+3)=6\times(3+3)$$

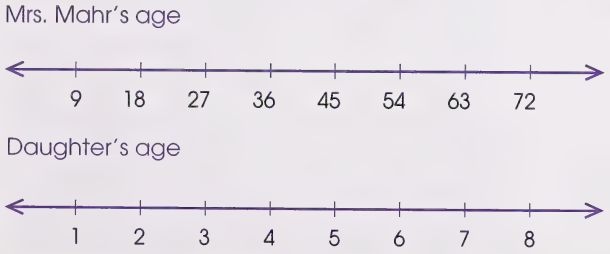
A number line shows the relationship between their ages now and in three years.



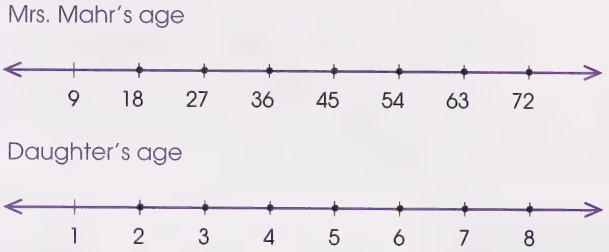
Once you understand the age problems, you can solve the original problem in several ways.

Method 1: Guessing, Checking, and Revising

Use logic to help make your guess. Mrs. Mahr is nine times the age of her daughter. The possible ages can be shown on a number line.



Mrs. Mahr's present age is probably greater than 18. So, the daughter's age is probably greater than 2. The following are possible ages for mother and daughter.

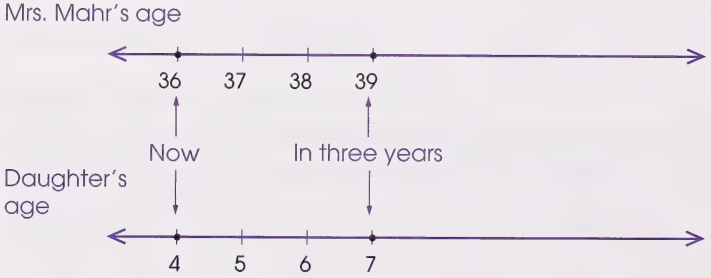


The problem states that Mrs. Mahr will be five times her daughter's age in three years.

What values will make this statement true?

Guess 1

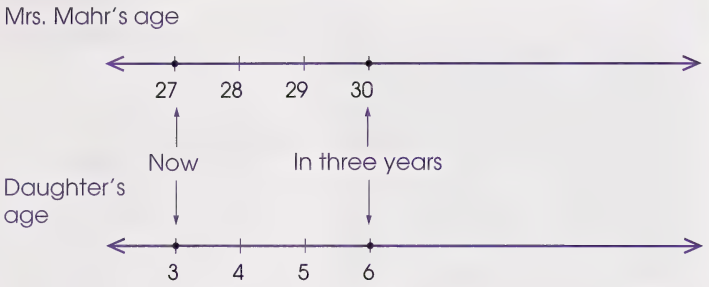
Mrs. Mahr is 36 and her daughter is 4. So, in three years Mrs. Mahr will be 39 and her daughter will be 7.



This guess is incorrect because $5\times 7\neq 39$.

Guess 2

Mrs. Mahr is 27 and her daughter is 3. So, in three years Mrs. Mahr will be 30 and her daughter will be 6.



This guess is correct because $5 \times 6 = 30$. This guess satisfies both conditions. Mrs. Mahr is nine times her daughter's age, and in three years she will be five times her daughter's age.

So, Mrs. Mahr is presently 27 and her daughter is 3.

Method 2: Using an Equation

Let the daughter's age now be n. Thus, Mrs. Mahr's age is $9n$.
In three years, the daughter's age will be $n+3$ and Mrs. Mahr's age will be $9n+3$.

You may wish to organize the data in a table.

	Present Age	Age in Three Years
Mrs. Mahr	$9n$	$9n+3$
Daughter	n	$n+3$

In three years, Mrs. Mahr will be five times older than her daughter.

Write an equation.

$$9n+3=5(n+3)$$

Solve the equation.

$$\begin{aligned} 9n+3 &= 5n+15 \\ 9n+3-3 &= 5n+15-3 \\ 9n &= 5n+12 \\ 9n-5n &= 5n+12-5n \\ 4n &= 12 \\ \frac{4n}{4} &= \frac{12}{4} \\ n &= 3 \end{aligned}$$

Now find Mrs. Mahr's age.

$$\begin{aligned} 9n &= 9 \times 3 \\ &= 27 \end{aligned}$$

Verification

In three years, will Mrs. Mahr be five times older than her daughter? Yes.

$$27 + 3 = 5(3 + 3)$$
$$30 = 5(6)$$
$$30 = 30$$

Therefore, Mrs. Mahr is now 27 years old and her daughter is 3 years old.

Solve the following problems. You may use any method.

1. Lori has twice as many dimes as she has quarters. If she has $9.00 in dimes and quarters, how many dimes does she have?

2. Mrs. Wong is presently nine years older than Mrs. Chan. In eight years, seven times Mrs. Chan's age will equal six times Mrs. Wong's age. What are their ages now?

3. A pair of track shoes costs twice as much as a pair of running shoes. If two pairs of track shoes and two pairs of running shoes cost $330, find the cost of each type.

PHOTO SEARCH LTD.

Check your answers by turning to the Appendix.

Rearranging Formulas Using Flow Charts

In this section you solved for a variable in a formula by rearranging the formula.

Another method used to solve formulas is to work backwards by using flow charts and reverse flow charts.

Remember: The operations of addition and subtraction are opposites, and the operations of multiplication and division are opposites.

Also, recall what you studied about flow charts and reverse flow charts in previous mathematics courses and in the section on problem-solving strategies in the Appendix of Module 1.

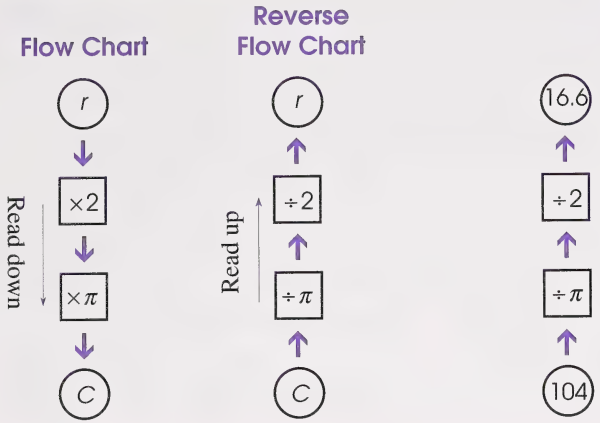
Flow charts include all steps that are normally used for solving equations, but you do not have to write out all the steps.

Example

Sonia wants to find the radius of a basketball. She measures the circumference and finds it to be 104 cm. She remembers one formula for circumference is $C = 2\pi r$. Use a flow chart and reverse flow chart to calculate the radius of the basketball.

Solution

You can represent the formula using a flow chart and reverse flow chart as follows:



The radius of the basketball is about 16.6 cm.

4. Use a flow chart and a reverse flow chart to find the value of the unknown variable in the following. Round your answer to one decimal.

a. $A = bh$ with $A = 28\ \text{cm}^2$ and $h = 4.5\ \text{cm}$

b. $A = \frac{1}{2}bh$ with $A = 74\ \text{cm}^2$ and $b = 16\ \text{cm}$

Check your answers by turning to the Appendix.

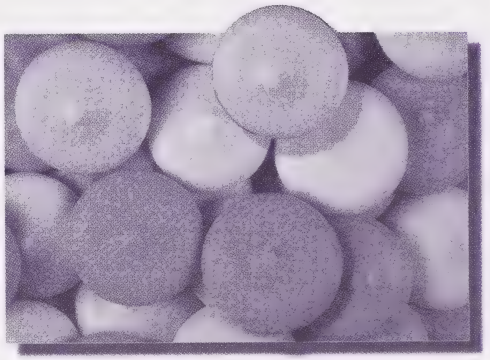
If you have access to a computer and the software *Understanding Equations* (Neufeld and Associates), you can use this program to review most of the topics in this section. This program is available for both Windows™ and Macintosh.

Enrichment

More Complicated Equations Involving Fractions

In this section you solved some simple equations containing fractions. Some equations containing fractions can be more complicated than others. These equations will require more steps to solve.

Example 1



Natalie, Sarah, and Georgina shared a package of mints. Natalie took one-half of the package, Sarah took one-quarter of the package, and Georgina took one-eighth of the package. If there were seven mints left after they took their shares, how many mints were in the package in the beginning?

Solution

Let n be the total number of mints in the package. Thus, Natalie took $\frac{1}{2}n\left(\text{or } \frac{n}{2}\right)$ mints, Sarah took $\frac{1}{4}n\left(\text{or } \frac{n}{4}\right)$ mints, and Georgina took $\frac{1}{8}n\left(\text{or } \frac{n}{8}\right)$ mints.

Write the equation.

$$\frac{n}{2}+\frac{n}{4}+\frac{n}{8}+7=n$$

Multiply by the LCM to clear the denominators.

$$8\left(\frac{n}{2}+\frac{n}{4}+\frac{n}{8}+7\right)=n\times 8$$

$$\overset{4}{\cancel{8}}\left(\frac{n}{\cancel{2}_{1}}\right)+\overset{2}{\cancel{8}}\left(\frac{n}{\cancel{4}_{1}}\right)+\overset{1}{\cancel{8}}\left(\frac{n}{\cancel{8}_{1}}\right)+8(7)=8n$$

$$4n+2n+n+56=8n$$

$$7n+56=8n$$

$$7n+56-7n=8n-7n$$

$$56=n$$

$$n=56$$

Verification

$$\frac{1}{2}\times 56=28,\qquad \frac{1}{4}\times 56=14,\qquad \frac{1}{8}\times 56=7$$

$$28+14+7+7=56$$

It checks out.

Therefore, there were 56 mints in the package in the beginning.

The following examples illustrate some of the different, more complicated equations involving fractions.

Example 2

Solve the equation $\frac{m+2}{4}=\frac{3m-6}{8}$.

Solution

Method 1

Write the equation.

$$\frac{m+2}{4}=\frac{3m-6}{8}$$

The LCM (lowest common multiple) of the denominators is 8. So, clear the denominators by multiplying both sides of the equation by 8; then simplify.

$$\cancel{8}^{2} \times \frac{m+2}{\cancel{4}_{1}} = \frac{3m-6}{\cancel{8}_{1}} \times \cancel{8}^{1}$$

$$2m+4 = 3m-6$$

$$2m+4-4 = 3m-6-4$$

$$2m = 3m-10$$

$$2m-3m = 3m-10-3m$$

$$-m = -10$$

$$m = 10$$

Method 2

Write the equation.

$$\frac{m+2}{4} = \frac{3m-6}{8}$$

Find the cross products.

$$\frac{m+2}{4} \quad \frac{3m-6}{8}$$

$$8(m+2) = 4(3m-6)$$

Simplify the equation.

$$8(m+2) = 4(3m-6)$$

$$8m+16 = 12m-24$$

$$8m+16-16 = 12m-24-16$$

$$8m = 12m-40$$

$$8m-12m = 12m-40-12m$$

$$\frac{-4m}{-4} = \frac{-40}{-4}$$

$$m = 10$$

Verification

LS	RS
$\frac{m+2}{4}$	$\frac{3m-6}{8}$
$= \frac{10+2}{4}$	$= \frac{3(10)-6}{8}$
$= \frac{12}{4}$	$= \frac{30-6}{8}$
$= 3$	$= \frac{24}{8}$
	$= 3$

LS = RS

The solution is $m = 10$.

Example 3

Solve the equation $\frac{2m}{3}+\frac{m}{4}=\frac{11}{6}$.

Solution

Write the equation.

$$\frac{2m}{3}+\frac{m}{4}=\frac{11}{6}$$

The LCM of the denominators is 12. Therefore, you can clear the denominators by multiplying both sides of the equation by 12 and simplifying.

$$12\left(\frac{2m}{3}+\frac{m}{4}\right)=\frac{11}{6}\times 12$$

$$\overset{4}{\cancel{12}}\left(\frac{2m}{\cancel{3}_{1}}\right)+\overset{3}{\cancel{12}}\left(\frac{m}{\cancel{4}_{1}}\right)=\frac{11}{\cancel{6}_{1}}\times\overset{2}{\cancel{12}}$$

$$8m+3m=22$$

$$11m=22$$

$$\frac{11m}{11}=\frac{22}{11}$$

$$m=2$$

Verification

LS	RS
$\frac{2m}{3}+\frac{m}{4}$	$\frac{11}{6}$
$=\frac{2(2)}{3}+\frac{2}{4}$	
$=\frac{4}{3}+\frac{2}{4}$	
$=\frac{16}{12}+\frac{6}{12}$	
$=\frac{22}{12}$	
$=\frac{11}{6}$	

LS = RS

1. Solve any **six** of the following equations by clearing the denominators or by using cross products.

a. $\frac{3m}{9}=\frac{7}{6}$

b. $\frac{y+2}{5}=\frac{2y+1}{9}$

c. $\frac{2d}{5}+\frac{d}{10}=\frac{15}{2}$

d. $\frac{2n}{5}=\frac{n-6}{4}$

e. $\frac{n}{2}+\frac{n}{3}=n-\frac{1}{8}$

f. $\frac{5}{n+3}=\frac{9}{2n}$

g. $\frac{a}{2}+\frac{a}{3}+\frac{a}{4}+\frac{a}{6}=15$

h. $\frac{5}{2(n+3)}=\frac{3}{n+4}$

2. When Mr. Johnson died, his estate was divided according to the instructions in his last will and testament. His wife Arlene was to receive one-half, his son Brad one-quarter, his niece Jill one-fifth, and the remainder was to be divided equally among his three favourite charities. If each charity received $3400, what was the total value of the estate and how much did each person receive?

3. Two-thirds of a number, decreased by one-half is equal to five-sixths. Find the number.

Check your answers by turning to the Appendix.

Equations that Contain Squares and Square Roots

There is only one two-digit number whose square root is equal to the sum of its digits. Do you know what it is?

$$\sqrt{ab} = a + b$$

You have worked with square roots in the previous module. In this section you solved some simple, first-degree equations. In this enrichment activity you will look at solving simple equations involving squares and square roots.

Remember as with any equation, you must maintain the balance by performing the same operation on each side of the equation.

Example 1

Solve $a^2 - 3 = 13$.

Solution

Add 3 to each side of the equation.

$$a^2 - 3 + 3 = 13 + 3$$
$$a^2 = 16$$

Take the square root of each side.

$$\sqrt{a^2} = \sqrt{16}$$
$$a = 4 \text{ or } -4$$

Remember: The square root of a number has two solutions: one positive and one negative. The square root of 16 is 4 or -4.

Verification

LS	RS
$a^2 - 3$	13
$= 4^2 - 3$	
$= 16 - 3$	
$= 13$	

LS = RS

LS	RS
$a^2 - 3$	13
$= (-4)^2 - 3$	
$= 16 - 3$	
$= 13$	

LS = RS

Therefore, $a = 4$ and $a = -4$ are both solutions to the equation.

Example 2

Solve $\sqrt{m} + 3 = 5$.

Solution

Subtract 3 from each side of the equation.

$$\sqrt{m} + 3 - 3 = 5 - 3$$
$$\sqrt{m} = 2$$

Square each side of the equation.

$$\left(\sqrt{m}\right)^2 = 2^2$$
$$m = 4$$

The square root symbol $\left(\sqrt{\ }\right)$ indicates the positive square root so there is only one value for m.

Verification

LS	RS
$\sqrt{m} + 3$	5
$= \sqrt{4} + 3$	
$= 2 + 3$	
$= 5$	

LS = RS

Therefore, $m = 4$.

Example 3

Solve $3\sqrt{a} = 6$.

Solution

Multiply each side of the equation by $\frac{1}{3}$ or divide by 3.

$$3\sqrt{a} = 6 \quad \text{or} \quad 3\sqrt{a} = 6$$
$$\cancel{3}^{1}\sqrt{a} \times \frac{1}{\cancel{3}_{1}} = \cancel{6}^{2} \times \frac{1}{\cancel{3}_{1}} \qquad \frac{3\sqrt{a}}{3} = \frac{6}{3}$$
$$\sqrt{a} = 2 \qquad \sqrt{a} = 2$$

Square each side of the equation.

$$\left(\sqrt{a}\right)^2 = 2^2$$
$$a = 4$$

Verification

LS	RS
$3\sqrt{a}$	6
$=3\sqrt{4}$	
$=3\times 2$	
$=6$	

LS = RS

Therefore, $a=4$.

4. Solve the following equations. Do at least **four** of the following.

a. $x^2+1=10$

b. $r^2-1=24$

c. $3t^2=108$

d. $-2n^2=-128$

e. $\sqrt{m}+2=8$

f. $5\sqrt{n}=10$

g. $25x^2=64$

h. $4b^2-3=33$

5. Write an equation that describes the situation; then solve. Do at least **two** of the following.

a. The square root of a number, plus three, is twelve. What is the number?

b. The square of a number, less eight, is seventeen. What is the number?

c. Ten times the square of a number is forty. What is the number?

d. Three times the square root of a number is fifteen. What is the number?

6. The mass of a freshly caught fish can be found by the following formula, where m is mass (in kg), ℓ is length (in cm), and g is girth (in cm).

$$m=\frac{\ell\times g^2}{24\,600}$$

Girth is the distance around the thickest part of the fish.

What is the mass of a salmon that is 82 cm long with a girth of 50 cm?

7. The approximate braking distance for a car on dry pavement is given by the following formula.

$$d=\frac{v^2}{210}$$

In this formula, d is the braking distance (in m) and v is the rate of speed (in km/h).

a. Find the approximate braking distance at 80 km/h. Round your answer to one decimal place.

b. The police officer who arrived at the scene of a car accident measured the skid marks and found that the braking distance was 62 m. Calculate the speed of the vehicle.

8. The time required for the swing of a pendulum is given by the following formula.

$$t = 0.2\sqrt{\ell}$$

In this formula, t is the time (in s) and ℓ is the length of the pendulum (in cm).

a. Find the time required for the swing of a pendulum that is 64 cm long.

b. Find the length of a pendulum that makes one swing in 2 s.

Check your answers by turning to the Appendix.

Conclusion

In this section you identified different ways to find the solutions to a variety of equations. You also discovered how to express various problem situations using equations and how to solve these equations in order to find the answers to the problems. Now that you are more familiar with the language of algebra and know how to apply it, you should feel comfortable tackling many more problems.

While you may not have learned how to communicate using sign language in this section, you did, however, communicate more fluently and solved problems more readily using the symbols and language of algebra.

Assignment

Assignment Booklet

You are now ready to complete the assignment for Section 1.

Section 2: Inequations

What does the sign in the photograph say? What does it mean? Some people interpret it to be a maximum, some a minimum, some a guideline. In reality, it is a prescribed maximum speed limit established for safe travel on that particular section of road—a limit that is determined by road design, traffic volume, and location.

Many situations that you come in contact with every day have prescribed limits in them. You must learn to understand and be guided by these limits (whether they be maximums, minimums, or a combination of both).

In mathematics, these limits are described using inequalities. In this section you will recognize the meaning of different inequalities, discover how to solve various inequations, and discover how to use inequations to describe and solve problem situations.

Activity 1: Modelling and Interpreting Inequations

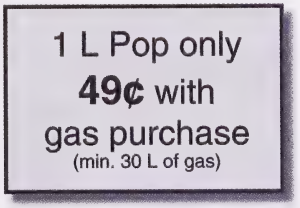
Good quarterbacks will complete at least 50% of their passes during the season.

Mathematical inequalities appear in a large number of situations in real life. Some other examples are depicted by the signs in the following question.

1. What do each of the following signs say? What does it mean?

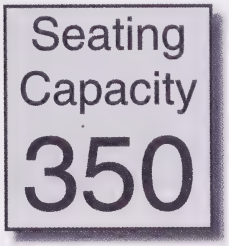
a.

b.

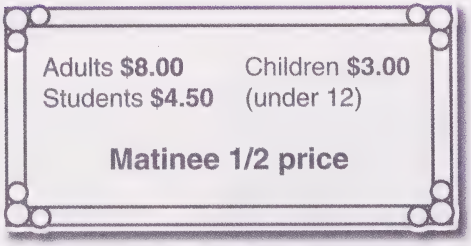


c.

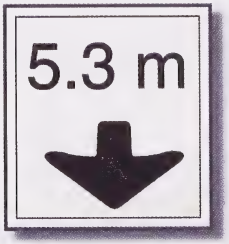
d.



e.



f.



g.

2. Does the sign in question 1.a. say you have to go 80 km/h? Can you go any speed under 80 km/h? Can you go over 80 km/h?

3. What is the maximum in question 1.b.?

4. Using the information from question 1.e., find the price of admission for a movie matinee for the following people.

   a. an adult **b.** a 12-year-old **c.** an 11-year-old

5. In question 1.g., is everything on sale for 70% off?

Check your answers by turning to the Appendix.

The signs in question 1 are all examples of inequalities. Limits are present in each one. Mathematical inequalities appear in many situations in the real world. You are probably aware of many more examples of inequalities.

The symbols for inequality are $\neq$, $>$, $<$, $\geq$, and $\leq$. These symbols each have a special meaning.

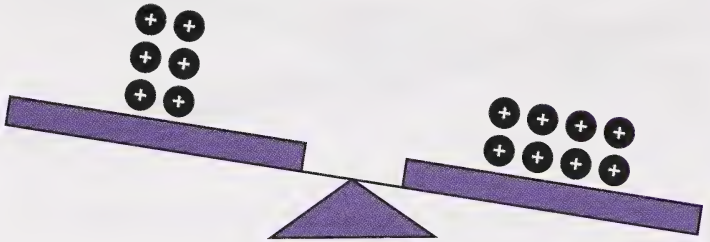
- $\neq$ means "is not equal to."
- $>$ means "is greater than."
- $<$ means "is less than."
- $\geq$ means "is greater than or equal to."
- $\leq$ means "is less than or equal to."

An **inequation** is a mathematical statement in which two expressions are connected by a symbol of inequality.

A scale can be used to model an inequation. Look at the following:

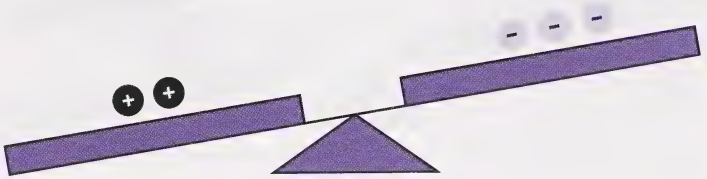
- $6 < 8$ is an inequation.

  It can be modelled as follows:



- $2 > -3$ is an inequation.

  It can be modelled as follows:



6. Which side has the lesser value in the inequation $6 < 8$?

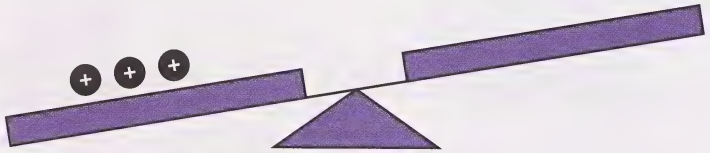
7. Which side has the greater value in the inequation $2 > -3$?

Check your answers by turning to the Appendix.

Here are a few more models of inequations.

- $3 > 0$ is an inequation.

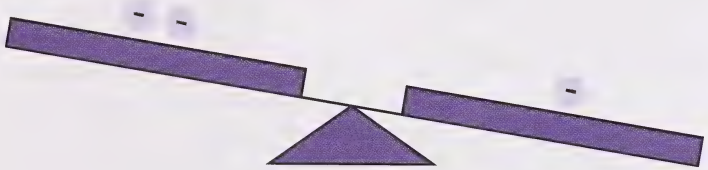
  It can be modelled as follows:



  Notice that the value of the left side is greater than the value of the right side.

- $-2 < -1$ is an inequation.

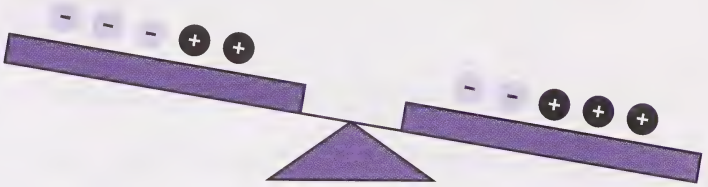
  It can be modelled as follows:



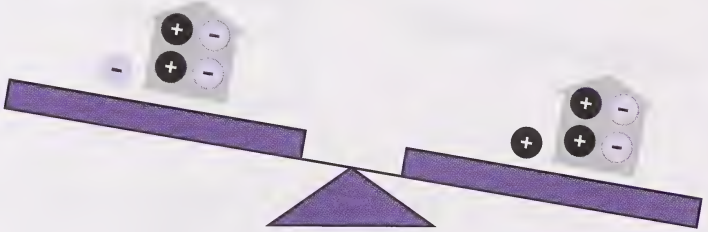
  Notice that the value of the left side is less than the value of the right side.

- $-3 + 2 < -2 + 3$ is an inequation.

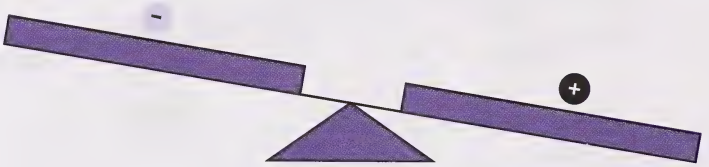
  It can be modelled as follows:



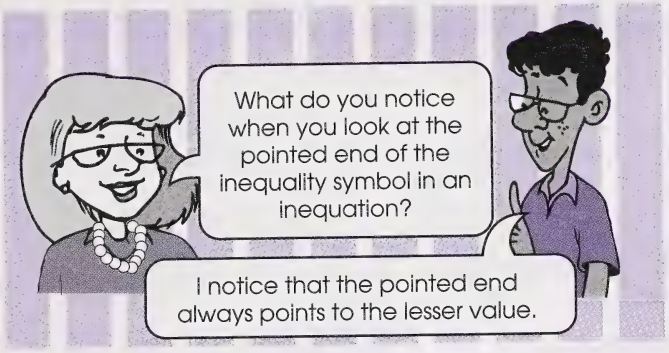
  Simplify the inequation by removing the zeros.



  The result is as follows:



  The left side has a value of -1. The right side has a value of $+1$. The inequation can be simplified to $-1 < +1$.


What do you notice when you look at the pointed end of the inequality symbol in an inequation?
I notice that the pointed end always points to the lesser value.

$$-1 < +1$$

lesser value — greater value

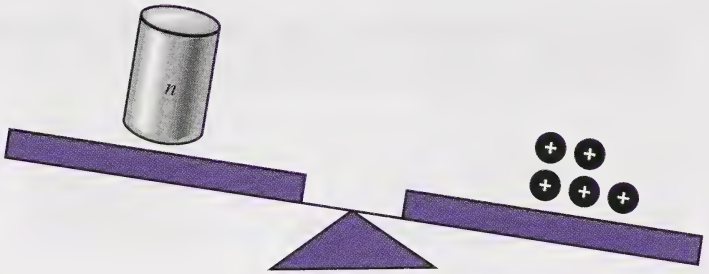
Inequations can also contain variables. Examine the following examples.

Example

Solve $n < 5$.

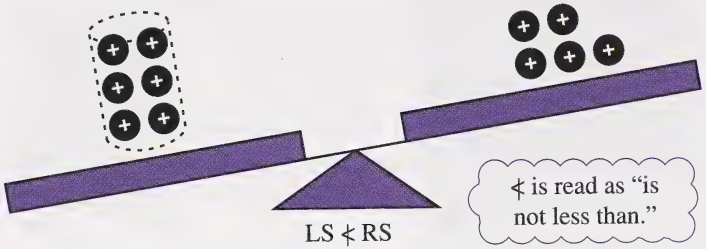
Solution

First model the inequation.



Is $n = 6$?

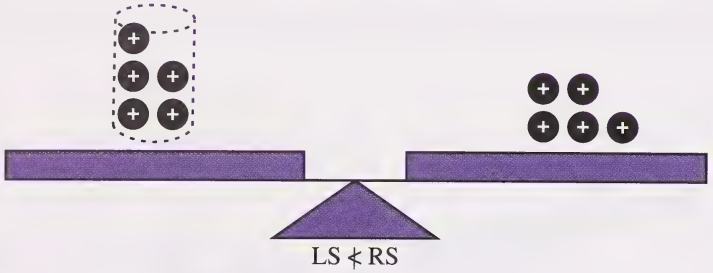
No. If $n = 6$, then the left side will not be less than the right side.



Therefore, $n \neq 6$.

Is $n = 5$?

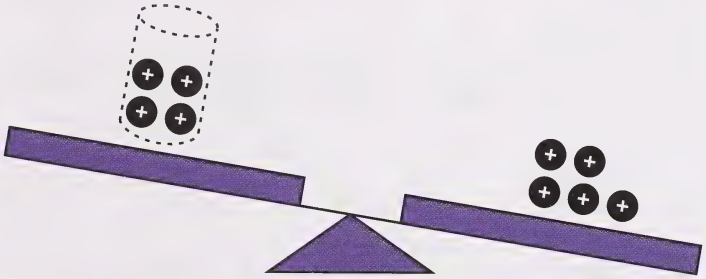
No. If $n = 5$, then the left side will not be less than the right side.



Therefore, $n = 5$ is not a solution.

Is $n = 4$?

Yes. If $n = 4$, then the left side will be less than the right side.

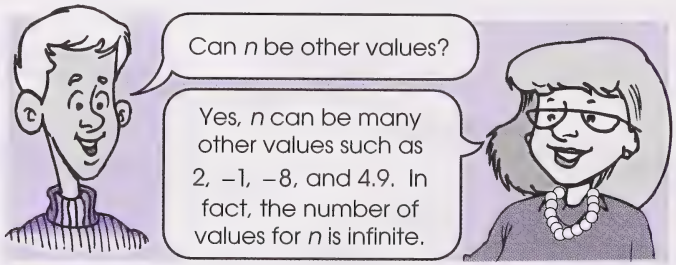


Therefore, $n = 4$. However, $n = 4$ is not the only solution.

Is $n = 3$?

Yes. If $n = 3$, then the left side will be less than the right side.

Therefore, $n = 3$.



8. Using the cut-out learning aids from the Appendix, model the following inequations.

a. $-6 > -8$ **b.** $n < -6$

c. $n + 2 < 5$ **d.** $3n > 6$

9. Write an inequation to express each of the following.

a. Junior boys are 12 years old and younger.

b. The legal drinking age is 18 years and older.

c. Bread is on sale for 69¢ per loaf. Limit 6 per customer.

d. Less than 4 cm of snow fell.

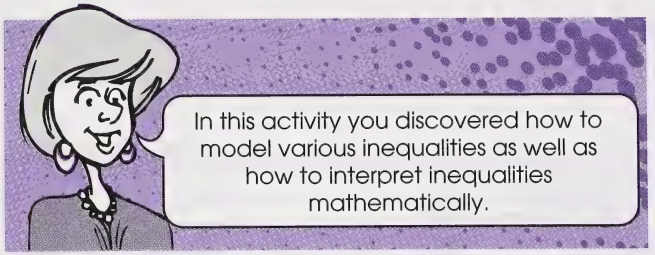
e. There were over 23 000 people at the football game.

f. Jack gained 2 kg last week and now weighs over 40 kg.

g. Warren earned $6.50 on the weekend but still doesn't have enough money to buy the $35 watch he wants.

h. Wendy sold three pairs of earrings and a $25 necklace. She made over $52 in total.

Check your answers by turning to the Appendix.


In this activity you discovered how to model various inequalities as well as how to interpret inequalities mathematically.

Activity 2: Solving Inequations Involving Variables with a Coefficient of One

Hank is a jockey at Edmonton Northlands race track. In a particular race the minimum mass of horse and jockey is 530 kg. If the horse has a mass of 481 kg and the saddle has a mass of 8 kg, what is the minimum mass Hank can be on the day of the race?

A question such as the previous one can be solved using an inequation with a variable that has a coefficient of one.

In Section 1 you discovered that adding or subtracting the same number on both sides of an equation will not change the equality. In this activity you will discover that the same property can be applied to inequations. Adding or subtracting the same number on both sides of an inequation will not change the inequality.

Example 1

Dennis scored 40 goals for his bantam hockey team before Christmas. During the holidays, they played in a tournament in Red Deer. If Dennis was still short of the 50 goal mark after the tournament, what is the greatest number of goals he could have scored in the tournament?

Solution

You can solve this problem by setting up an inequation.

Let n be the number of goals Dennis could have scored in the tournament.

Write the inequation.

$$n + 40 < 50$$

Isolate the variable by subtracting 40 from each side.

$$n + 40 - 40 < 50 - 40$$
$$n < 10$$

Verification

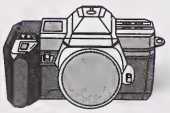
Verify the solution with any value less than 10. Try $n = 9$.

LS	RS
$n + 40$	50
$= 9 + 40$	
$= 49$	

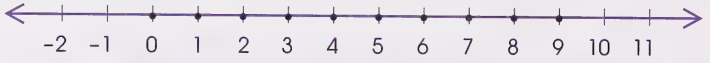
LS < RS

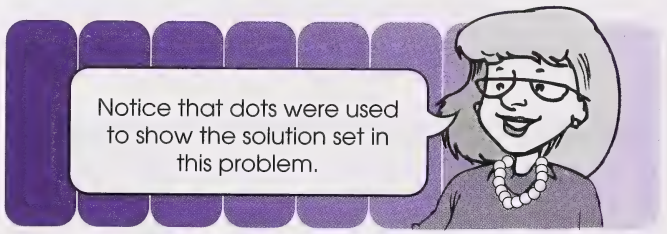
Therefore, the solution is $n < 10$.

A picture is worth a thousand words. Undoubtedly, you have heard this expression before. What it means of course is that a picture usually tells the story better than a word description. That is why plotting a solution of an inequation on a number line gives a clear picture of the numbers that are possible solutions.



For instance, the solution to Example 1 can be shown on a number line as follows:





1. Use the information in Example 1 and in the previous number line to answer the following questions.

   a. Why do you think dots were used in the number line?

   b. Is zero a possible part of the solution?

   c. Is -1 a possible part of the solution? Explain.

   d. Is 7.5 a possible solution? Explain.

   e. State the solution set in words.

Check your answers by turning to the Appendix.

Now you can analyse the solutions of some general inequations.

Example 2

Solve the inequation $k-1<12$.

Solution

$$k-1<12$$
$$k-1+1<12+1 \quad \leftarrow \text{Isolate the variable by adding 1 to both sides.}$$
$$k<13$$

Verification

Verify the solution with any value less than 13. Try $k=12$.

LS	RS
$k-1$	12
$=12-1$	
$=11$	

LS < RS

Thus, the solution is $k<13$.

The solution to Example 2 can be shown on a number line as follows:

2. Use the previous number line to answer the following questions.

   a. Why is a solid line drawn to the left of 13 rather than dots?

   b. Why is the value 13 shown with an open circle?

Check your answers by turning to the Appendix.

Example 3

Solve the inequation $m+4 \geq -2$.

Solution

$$m+4 \geq -2$$
$$m+4-4 \geq -2-4 \quad \leftarrow \text{Subtract 4 from both sides.}$$
$$m \geq -6$$

Verification

Try $m=-6$.

LS	RS
$m+4$	-2
$=(-6)+4$	
$=-2$	

LS = RS

Try $m=-5$ (a value greater than -6).

LS	RS
$m+4$	-2
$=(-5)+4$	
$=-1$	

LS > RS

Thus, $m \geq -6$ is the correct solution.

The solution to Example 3 can be shown on a number line as follows:

3. Use the previous number line to answer the following questions.

a. Why is there a solid line to the right of -6?

b. Why is the value -6 shown with a solid dot?

4. Solve each of the following inequations. Be sure to verify the solutions and show the solution on a number line.

a. $m+7<10$ **b.** $m-2>11$ **c.** $x+4 \leq 6$

d. $y-3>8$ **e.** $w+2>6$ **f.** $n+7 \geq -4$

5. Does the indicated replacement for the variable make the inequation true?

a. $n+7 \leq -2 \quad (n=-7)$ **b.** $m-2>-4 \quad (m=1)$

c. $w+5 \geq 0 \quad (w=-5)$ **d.** $y-4<-5 \quad (y=-4)$

e. $p+4<7 \quad (p=1.5)$ **f.** $t-1>-2 \quad (t=-1)$

Solve the following problems by writing inequations that describe the situation and then solving. Display the solution on a number line.

6. Jeremy collects hockey cards. During a show last weekend he purchased 15 more cards. He now has over 110 cards.

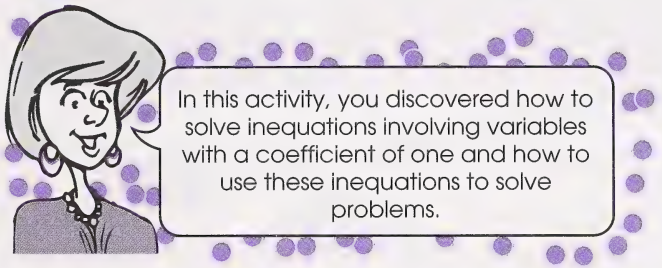
   a. How many did he have before?

   b. What is the largest number he could have had before last weekend?

7. A number increased by 4 is greater than or equal to 7. Find the number.

8. Mr. Cartwright sold 35 head of his cattle at the auction market on Wednesday. He still has over 250 head of cattle. How many did he have before Wednesday?

Check your answers by turning to the Appendix.


In this activity, you discovered how to solve inequations involving variables with a coefficient of one and how to use these inequations to solve problems.

Activity 3: Solving Inequations Involving Variables with Coefficients Other than One

You have more than twice as many fish as your friend Sal. If you have 10 fish, how many can your friend Sal have?

This problem, and others like it, can be solved using an inequation in which the coefficient of the variable is a number other than one. In this activity you will discover how to solve an inequation of this type.

In Section 1 you were shown that you can multiply or divide both sides of an equation by the same number and still keep the equality.

Can you solve inequations using the same methods that were used when solving equations? As you answer questions 1 to 5, look for any relationships there might be between solving equations and inequations.

1. Copy and complete the following table into your notebook. The first two have been done for you.

Inequation	Operation to Both Sides	New Inequation	True or False
$-6<2$	Add 2.	$-4<4$	T
$-6<2$	Add -2.	$-8<0$	T
$-6<2$	Subtract 2.		
$-6<2$	Subtract -2.		
$-6<2$	Multiply by 2.		
$-6<2$	Multiply by -2.		
$-6<2$	Divide by 2.		
$-6<2$	Divide by -2.		

2. After which operations was the new inequation still true?

3. After which operations was the new inequation false?

4. Check this out by repeating the same operations on the inequation $8>6$.

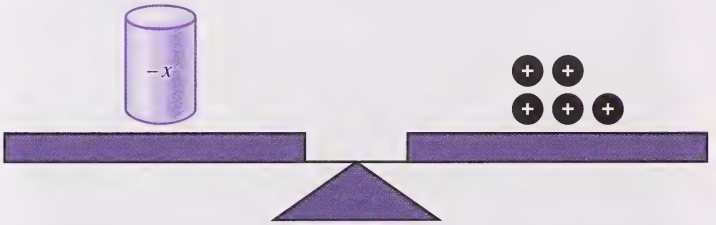
5. How could you change those inequations that were false so that they would be true?

Check your answers by turning to the Appendix.

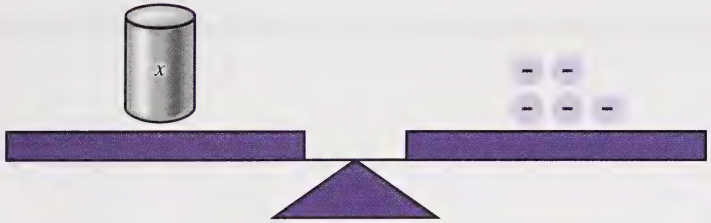
What happens if you model the solution of an inequation?

In the previous section you solved equations like $-6x+8=20$ and $4-2x=x+13$. You discovered that multiplying or dividing each side of an equation by a negative number does not change the equality.

For example, $-x=5$ can be modelled as follows:



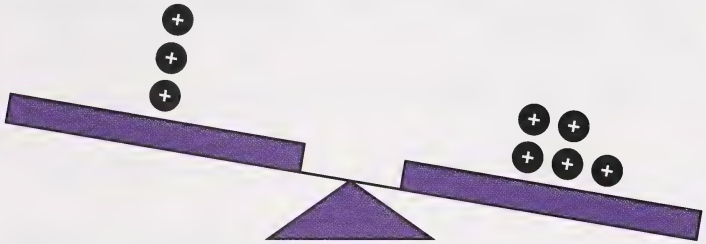
To solve $-x=5$, you can multiply each side by -1 (or replace each side with its inverse).



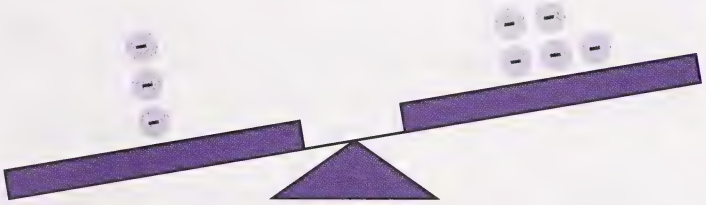
So, $x = -5$.

Notice that if you multiply each side of an inequation by -1, the inequality is **not** maintained. Examine the following:

- $3 < 5$ is an inequation. It can be modelled as follows:

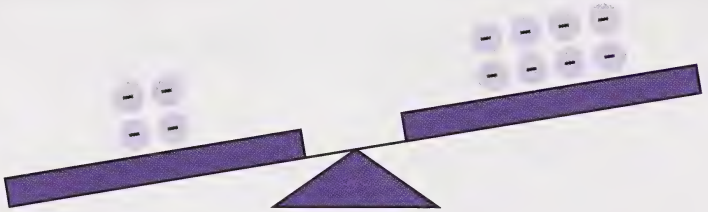


If you multiply each side by -1 (or replace each side with its inverse), the inequality changes.

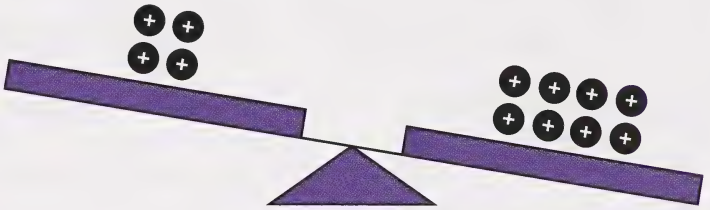


$-3 > -5$

- $-4 > -8$ is an inequation. It can be modelled as follows:



If you multiply each side by -1 (or replace each side with its inverse), the inequality changes.



$4 < 8$

6. What does this tell you about solving inequations?

Check your answer by turning to the Appendix.

When you multiply or divide both sides of an inequation by a negative number, you must reverse the sign of inequality. Greater than (>) becomes less than (<) and vice versa.

Example 1

Solve the inequation $5x \geq 30$. Display the solution on a number line.

Solution

Isolate the variable by dividing by 5.

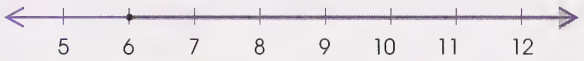
$$5x \geq 30$$
$$\frac{5x}{5} \geq \frac{30}{5}$$
$$x \geq 6$$

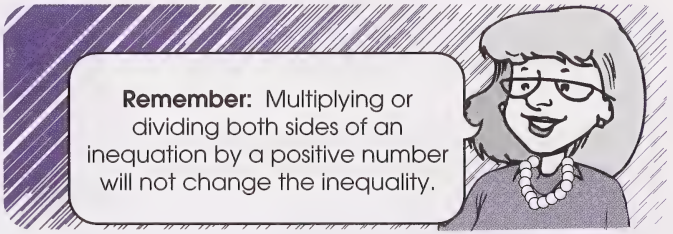
Verification

LS	RS
$5x$	30
$=5(6)$	
$=30$	
LS $\geq$	RS

The solution is $x \geq 6$.

The solution to Example 1 can be shown on a number line as follows:





Example 2

Solve the inequation $-6m > 24$.

Solution

Isolate the variable by dividing both sides of the inequation by -6.

$$-6m > 24$$
$$\frac{-6m}{-6} < \frac{24}{-6}$$
$$m < -4$$

Since both sides are divided by a negative number, the sign of the inequality must be reversed.

Verification

Let $m = -5$ (a number less than -4).

LS	RS
$-6m$	24
$= -6(-5)$	
$= 30$	

LS > RS

Therefore, $m < -4$ is the solution.

7. Use the information from Example 2 to answer the following questions.

a. Could $m = -4$ be a solution for the inequation? Explain.

b. Could $m = -4.1$ be a solution? Explain.

c. Display the solution on a number line.

8. What number would you divide both sides of each inequation by in order to isolate the variable?

a. $4m < -8$ **b.** $-2m \leq -8$ **c.** $1.5m > 37.5$

9. Solve each of the following inequations. Verify your solutions, and display each solution on a number line.

a. $4m < 12$ **b.** $3m < 27$ **c.** $-5b > 30$

d. $-4b \geq 10$ **e.** $2.4b < 8.4$

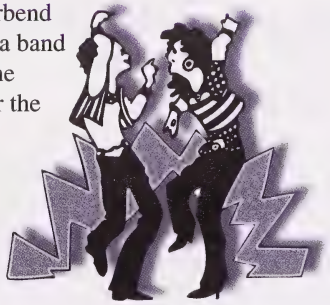
10. Write an inequation that describes each of the following situations; then solve the inequation. Display each solution on a number line.

a. Multiplying a number by 8 results in a product that is less than 44. Find the number.

b. Jason works part-time as a cashier for a fast-food outlet, where he makes $6.25 an hour. He wants to make more than $280 over the next three weeks. What is the least number of hours he must work in order to achieve his goal? (Assume Jason has to work to the nearest whole hour.)

c. A movie theatre is having six special showings of a film on Saturday and Sunday. They're offering it at a special price of $2.25 per person. In order to meet all their expenses for the six shows, they need to have an income of at least $1500.00 from ticket sales. What is the least number of tickets they need to sell to achieve this target?

d. The student council at Riverbend Junior High decided to hire a band for the Christmas dance. The band and the decorations for the gym cost $750.00. The student council is going to charge $3.25 per dance ticket, and they are going to donate any profits made to the Christmas Bureau.



i. What is the minimum number of tickets that need to be sold in order to cover the cost of the band and the decorations?

ii. What is the minimum number of tickets that need to be sold if the student council wants to have at least $50 to give to the Christmas Bureau?

11. Helia wants to keep her annual business travel expenses under $10 000. Assuming Helia takes four weeks off every year, how much can Helia spend each week? Round your answer to the nearest dollar.

Check your answers by turning to the Appendix.

Some inequations may involve fractions.

Example 3

Solve the inequation $\frac{a}{6} \leq -12$.

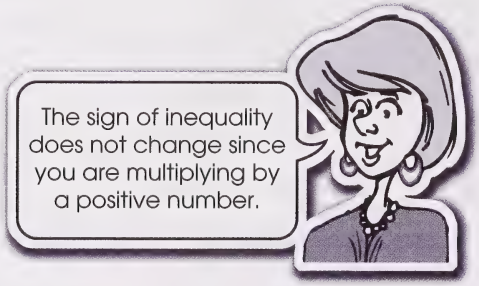
Solution

Isolate the variable by multiplying both sides of the inequation by 6.

Remember: $\frac{a}{6}$ means the same as $\frac{1}{6}a$.

$$\frac{a}{\cancel{6}_{1}} \times \cancel{6}^{1} \leq -12 \times 6$$

$$a \leq -72$$



Verification

Let $a = -78$ (a number less than -72 and divisible by 6).

LS	RS
$\frac{a}{6}$	-12
$= -78 \div 6$	
$= -13$	

LS $\leq$ RS

Therefore, $a \leq -72$ is the solution.

12. Use the information from Example 3 to answer the following questions.

a. Could $a = -72$? Explain.

b. Could $a = -75$? Explain.

c. How many possibilities are there for a?

d. Display the solution on a number line.

Check your answers by turning to the Appendix.

Example 4

Solve the inequation $\frac{m}{-3} > 9$.

Solution

$$\frac{m}{-3} > 9$$

$$\frac{m}{\cancel{-3}_{1}} \times \left(\frac{1}{\cancel{-3}}\right) < 9 \times (-3)$$

$$m < -27$$

Remember: Reverse the inequality symbol.

Verification

Let $m = -30$.

LS	RS
$\frac{m}{-3}$	9
$= -30 \div (-3)$	
$= 10$	
LS $>$	RS

So, $m < -27$ is the solution.

13. Use the information from Example 4 to answer the following questions.

a. Why were both sides multiplied by -3?

b. Why did the sign of inequality change when both sides were multiplied by -3?

14. What number would you multiply both sides of the following inequations by in order to isolate the variable?

a. $-\frac{1}{3}m \geq 2$ **b.** $\frac{m}{2} > 5$ **c.** $\frac{2}{3}m < \frac{5}{6}$

15. Solve each of the following inequations. Verify your solutions, and display each solution on a number line.

a. $\frac{3}{4}b > \frac{15}{16}$ **b.** $-\frac{b}{2} \leq -4$

16. Alexis obtained 78%, 82%, and 88% on her first three tests. What must her fourth mark be in order to obtain an average of at least 85%?

17. The quotient of a number and 5 is less than -12. Find the largest integer that satisfies this condition.

18. Nathan has $85 to buy a new jacket. The store where he is planning to buy the jacket has a sale of 40% off. Calculate the highest, regular-priced jacket that Nathan can afford. Round your answer to the nearest cent.

Check your answers by turning to the Appendix.

Now Try This

Solve the following challenge problems.

The purpose of this exercise is to make you think. Can you make the connection?

These are quarks.	These are not quarks.
madam	complete
yearly	wagon
hearth	butterfly
nomination	heavy
synthesis	nothing
tent	forever

19. Which of the following are quarks? Why?

catastrophic, forget, slanted, algebra, level, calm, remember, arithmetic, soothing, hilly

20. Use the table to answer the following questions.

These are quimps.	These are quacks.	These are neither quimps nor quacks.
H F E M N	A L X T Y	B G O P Q

a. Which of the following are quimps? Which are quacks?

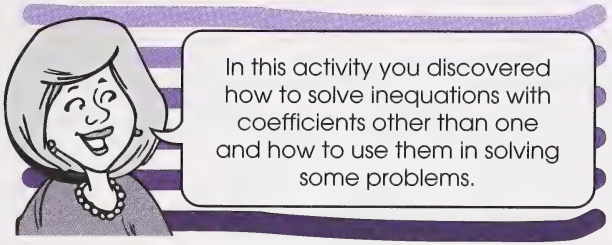
I, K, W, U, V, D

b. What do quimps and quacks have in common?

c. What makes quimps different from quacks?

d. Why are B, G, O, P, and Q neither quimps nor quacks?

Check your answers by turning to the Appendix.


In this activity you discovered how to solve inequations with coefficients other than one and how to use them in solving some problems.

Activity 4: Solving Multi-Step Inequations

There is a limit to the amount of weight that a skytram can safely carry. This weight includes any equipment (such as backpacks) plus the average weight of each passenger times the number of passengers. How many passengers can one car of a sky tram carry if they have equipment weighing a total of 100 kg and the load limit per car is 550 kg?

PHOTO SEARCH LTD.

If you know the average weight of the passengers, you can solve this problem using an inequality.

Inequations involving several steps can be solved in exactly the same as solving equations involving several steps. But you must keep in mind the very important concept regarding inequations—multiplying or dividing both sides by a negative number requires that you reverse the sign of inequality—as you follow the steps in solving the inequation.

For instance, assume the average mass of a passenger is 73 kg.

Then the inequality is $73n + 100 \leq 550$.

Solve the inequality to determine the maximum number of passengers that can go in one car.

$$\begin{aligned} 73n + 100 &\leq 550 \\ 73n + 100 - 100 &\leq 550 - 100 \\ 73n &\leq 450 \\ \frac{73n}{73} &\leq \frac{450}{73} \\ n &\leq 6.164\,383\,562 \end{aligned}$$

Therefore, the maximum number of passengers the skytram car can carry safely is 6.

Follow these general inequations involving more than one step.

Example 1

Solve the inequation $4x - 3 < 17$.

Solution

$$\begin{aligned} 4x - 3 &< 17 \\ 4x - 3 + 3 &< 17 + 3 \quad \text{← To isolate } 4x\text{, add 3 to both sides.} \\ 4x &< 20 \\ \frac{4x}{4} &< \frac{20}{4} \quad \text{← To isolate } x\text{, divide both sides by 4.} \\ x &< 5 \end{aligned}$$

Verification

LS	RS
$4x - 3$	17
$= 4(4) - 3$	
$= 16 - 3$	
$= 13$	

LS < RS

Thus, $x < 5$ is the correct solution.

Example 2

Solve the inequation $-8n + 1 < 17$.

Solution

$$\begin{aligned} -8n + 1 &< 17 \\ -8n + 1 - 1 &< 17 - 1 \\ -8n &< 16 \\ \frac{-8n}{-8} &> \frac{16}{-8} \quad \text{← Remember to reverse the sign of inequality at this point.} \\ n &> -2 \end{aligned}$$

Verification

LS	RS
$-8n+1$	17
$=-8(-1)+1$	
$=8+1$	
$=9$	

LS < RS

Thus, $n>-2$ is the correct solution.

1. For each of the following inequations, state if you would need to reverse the sign of equality? For those that you stated yes, explain.

a. $4m<-18$ **b.** $m-2<7$ **c.** $-3n+6\geq 27$

d. $\frac{m}{4}+5>-2$ **e.** $\frac{1}{4}m-2\leq -4$ **f.** $-2.5m+1<7$

2. Write the new inequation that you would get as a result of performing the indicated operation(s) to both sides.

a. $-7b-6<-4$ (Add 6.)

b. $-3m\geq 9$ (Divide by -3.)

c. $\frac{m}{4}<-8$ (Multiply by 4.)

d. $-6m-5>7$ (Add 5; then divide by -6.)

e. $-4p+2\leq -14$ (Subtract 2; then divide by -4.)

f. $-1.5w-2>7$ (Add 2; then divide by -1.5.)

3. Solve each of the following inequations. Verify your answers.

a. $4m-2>-6$ **b.** $5-3d\leq 20$ **c.** $\frac{m}{2}+1\leq 3$

d. $-p+4>3$ **e.** $\frac{1}{4}t-5<1$ **f.** $-2.5m+8\leq -2$

Solve the following problems using inequations. Show your solution on a number line.

4. When an integer is tripled and then increased by four, the result is less than ten. Find the largest integer for which this is true. Find two integers for which it is not true.

5. Eleven adults and two children got on an elevator. The door closed but the elevator would not move because the load exceeded the safety limit of 1000 kg. If the weight of the two children together was 80 kg, what was the average weight of the adults?

6. Belinda has a mark of 84% in Science and 82% in Math. If her marks in Social Studies and Language Arts are identical, then what is the lowest mark they can be in order to achieve an honours standing (80% or higher average)? Display the solution on a number line.

7. Michael and Ian each have a small collection of antique model railway cars. Michael has five more than Ian. Double the number that Ian has plus the number Michael has is less than or equal to fourteen. What are the possibilities for the number of cars Ian has in his collection?

8. Mr. Larouche hires a taxi to give him a sightseeing tour of greater Montreal. The taxi charges a basic rate of $2.50 plus $1.50/km. Mr. Larouche doesn't want to spend more than $40.00. What is the farthest distance he can travel?

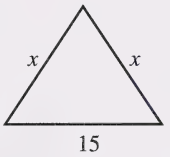
Check your answers by turning to the Appendix.

Now Try This

Use one of the problem-solving strategies to solve the following problem.

9. You are to make a triangle like the one given such that the perimeter is no more than 45 m.

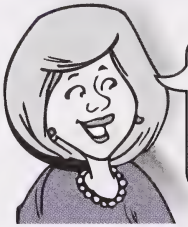
   a. Write and solve an inequation to find the possible values for x.



   b. For what whole number values of x is the triangle isosceles?

   c. For what whole number value of x is the triangle equilateral?

Check your answers by turning to the Appendix.



Having completed this activity, you are now able to solve multi-step inequations and to use them to solve certain problems.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

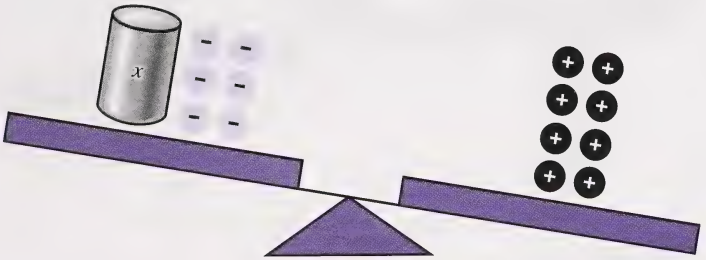
Earlier in this module you looked at solving equations using models. If you had any difficulty solving inequalities, you will use the same ideas to solve inequations using models.

Example 1

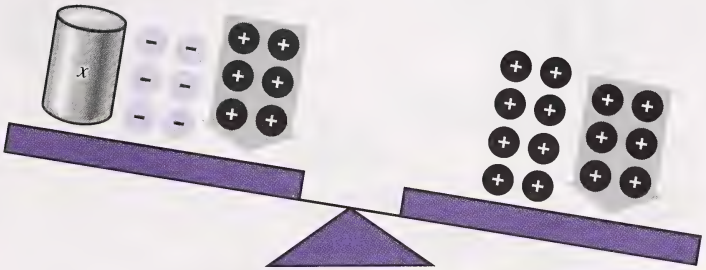
Solve the inequation $x - 6 < 8$.

Solution

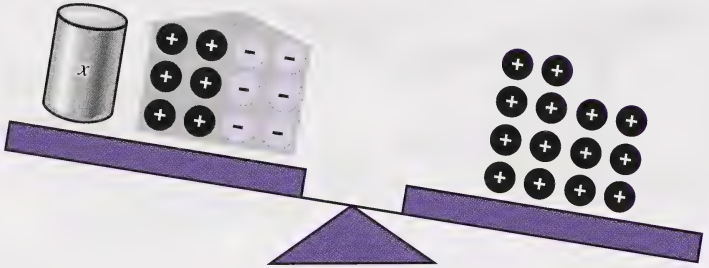
Model the inequation.



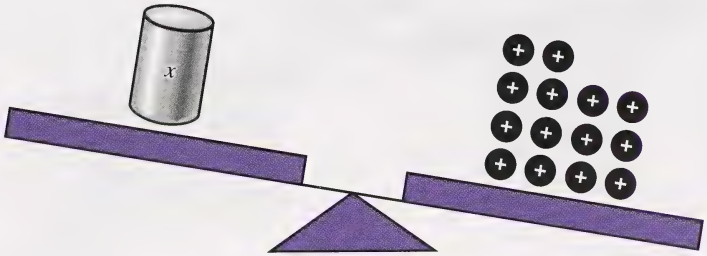
Add $+6$ (the additive inverse of -6) to both sides.



Simplify the inequation by removing the zeros.



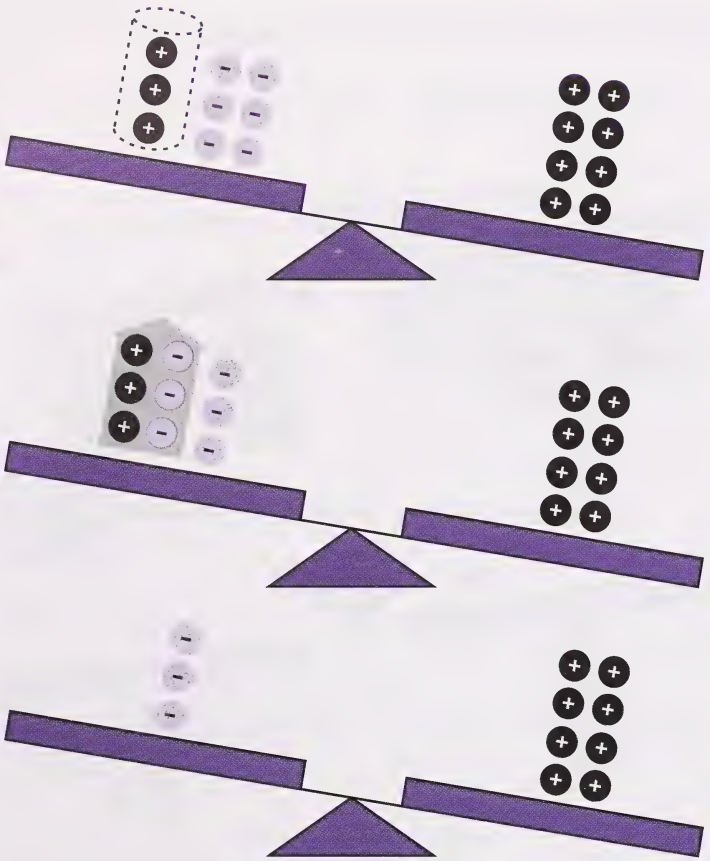
The result is this.



Therefore, $x < 14$.

Verification

Replace x with any value less than 14. Try $x = 3$.



Since $-3 < 8$, the statement $x - 6 < 8$ is true.

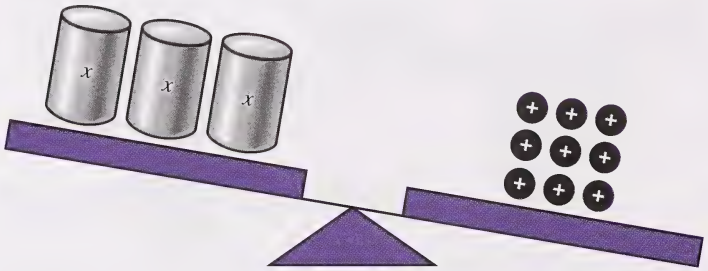
The solution to Example 1 can be shown on a number line as follows:

Example 2

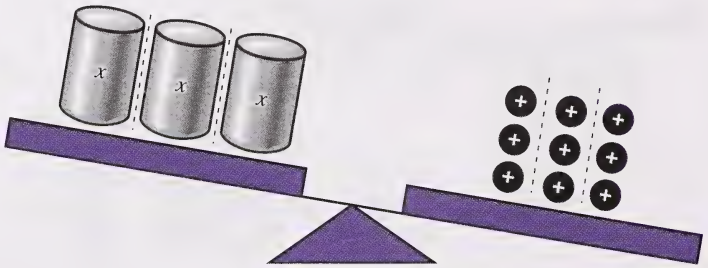
Solve the inequation $3x < 9$.

Solution

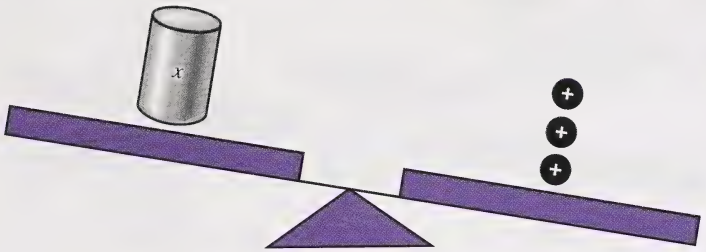
Model the inequation.



Divide each side into three groups.



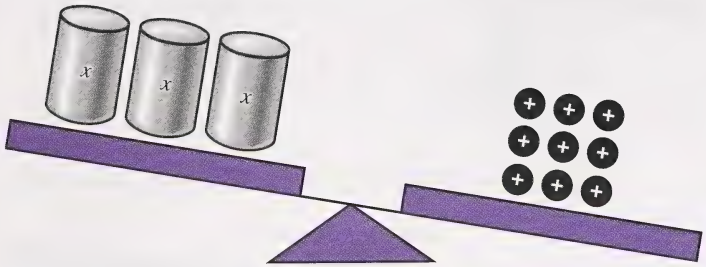
To solve the inequation, examine one group from each side.

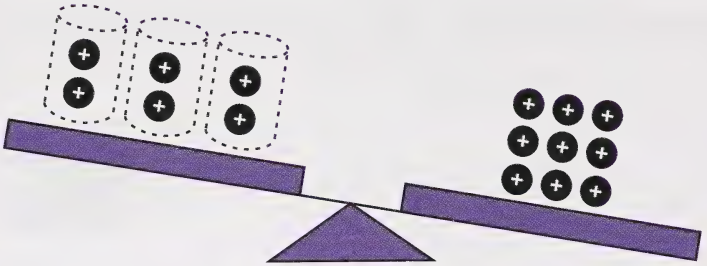


Therefore, $x < 3$.

Verification

Replace x with any value less than 3. Try $x = 2$.





Since $6 < 9$ then the statement $3x < 9$ is true when $x < 3$.

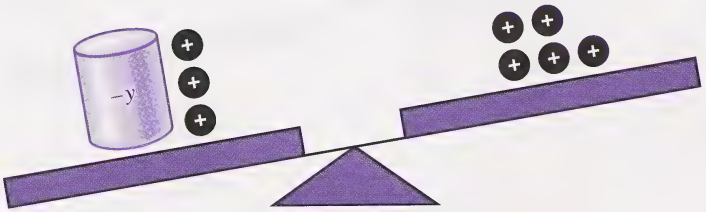
The solution to Example 2 can be shown on a number line as follows:

Example 3

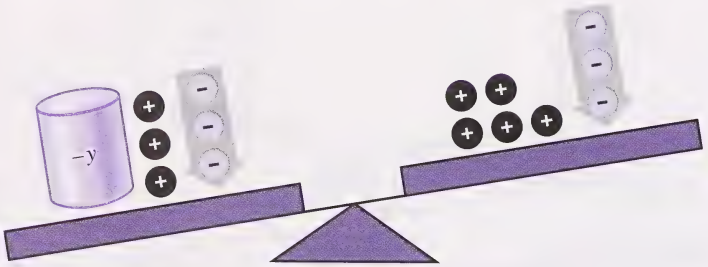
Solve the inequation $-y + 3 > 5$.

Solution

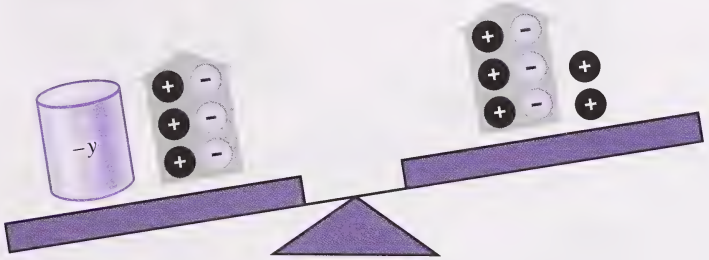
Model the inequation.



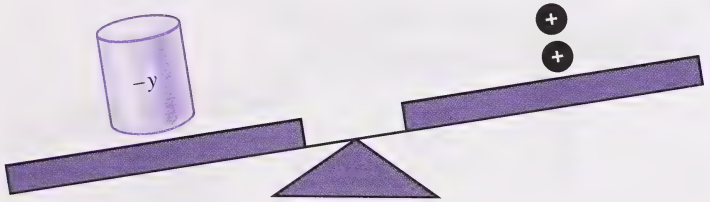
Add -3 (the additive inverse of $+3$) to each side.



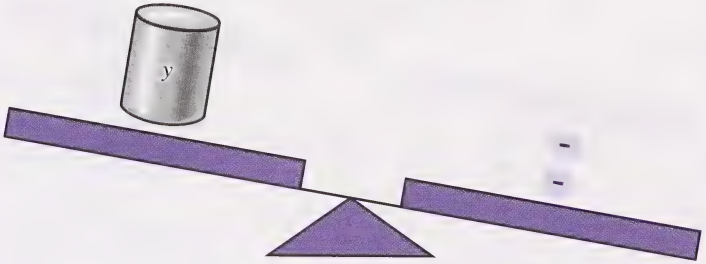
Simplify the inequation by removing the zeros.



The result is this.



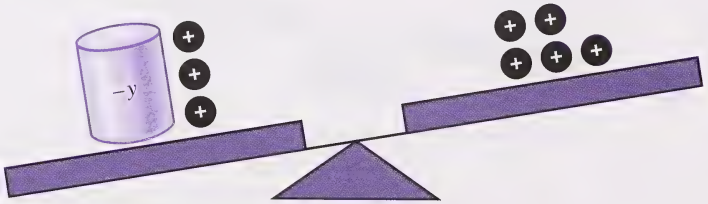
Now you need to change the $-y$ to a $+y$. Change the sign to the opposite on both sides. Also change the balance.



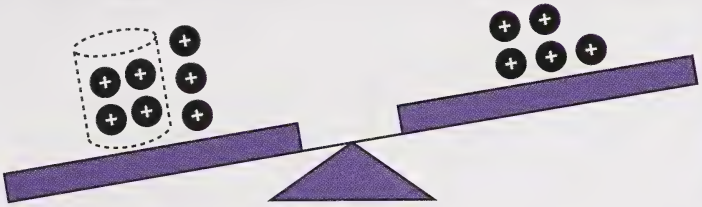
Therefore, $y < -2$.

Verification

Replace y with any value less than -2. Try $y = -4$.

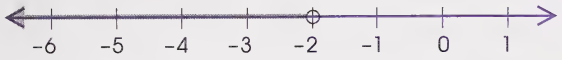


If y is -4, then $-y$ will be 4.



Since $7 > 5$, the statement $-y + 3 > 5$ is true when $y < -2$.

The solution to Example 3 can be shown on the number line as follows:



1. **a.** How does the model indicate that the sign in the inequality has changed?

b. How does the inequation show that the sign in the inequality has changed?

2. How does modelling inequations compare with modelling equations?

3. Use paper and pencil to solve the following inequations. Verify your solutions, and draw each solution on a number line. You may use the cut-out learning aids from the Appendix to help you solve these inequations.

a. $m + 7 < 10$ **b.** $m - 1 > 10$

c. $-4m > 12$ **d.** $-4x + 8 \leq 14$

e. $5m + 8 < -2$ **f.** $-0.75p - 1 < -4$

Check your answers by turning to the Appendix.

Enrichment

If you experienced no difficulties in solving the inequations in this section and you were able to solve the various types of equations in the previous section, you should experience no major problems when solving more complicated inequations. Study the following examples carefully before proceeding to the questions that follow.

Example 1

Solve the inequation $9a < 4a - 25$.

Solution

$$9a < 4a - 25$$
$$9a - 4a < 4a - 25 - 4a \quad \leftarrow \text{Subtract } 4a \text{ from both sides.}$$
$$5a < -25$$
$$\frac{5a}{5} < \frac{-25}{5} \quad \leftarrow \text{Divide both sides by 5.}$$
$$a < -5$$

Verification

LS	RS
$9a$	$4a-25$
$=9(-6)$	$=4(-6)-25$
$=-54$	$=-24-25$
	$=-49$

LS $<$ RS

The solution is $a<-5$.

Example 2

Solve the inequation $6-2b\geq 5b+20$.

Solution

$$6-2b\geq 5b+20$$

$$6-2b-6\geq 5b+20-6$$ ← Subtract 6 from both sides.

$$-2b\geq 5b+14$$

$$-2b-5b\geq 5b+14-5b$$ ← Subtract $5b$ from both sides.

$$-7b\geq 14$$

$$\frac{-7b}{-7}\leq\frac{14}{-7}$$ ← Divide both sides by –7.

$$b\leq -2$$

Verification

LS	RS
$6-2b$	$5b+20$
$=6-2(-3)$	$=5(-3)+20$
$=6-(-6)$	$=-15+20$
$=12$	$=5$

LS $\geq$ RS

The solution is $b\leq -2$.

1. Solve the following inequalities. Verify your answers.

a. $4(c-2)<12$

b. $3d+8>5d-6$

c. $7e+9\leq 5e-6$

d. $9-2f<4f-15$

e. $4g\geq 9g-35$

f. $6(n+2)>36$

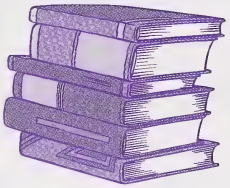
Check your answers by turning to the Appendix.

In Mathematics 9 you have worked with natural numbers, whole numbers, integers, and rational numbers.

When you were representing inequalities on number lines, you may have noticed that the line was continuous when you were asked to represent all numbers greater than or less than a given number. What other kinds of numbers are there? Additional numbers include the irrational numbers and the real numbers.

2. Using mathematics books in your local library or the Internet, obtain some information on irrational numbers, real numbers, and complex numbers.

Check your answer by turning to the Appendix.



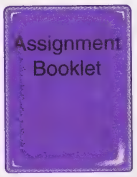
Conclusion

In this section you applied your experience with solving equations in Section 1 to solving inequations. You then used this knowledge to solve problems involving inequations. You also discovered how to represent solutions to inequations graphically on a number line as well as distinguish between the kinds of numbers acceptable as solutions.

You interpreted the speed limit sign at the beginning of this section and examined maximums and minimums. You now know that a maximum speed of 60 km/h means you can travel at a speed of 60 km/h or less.

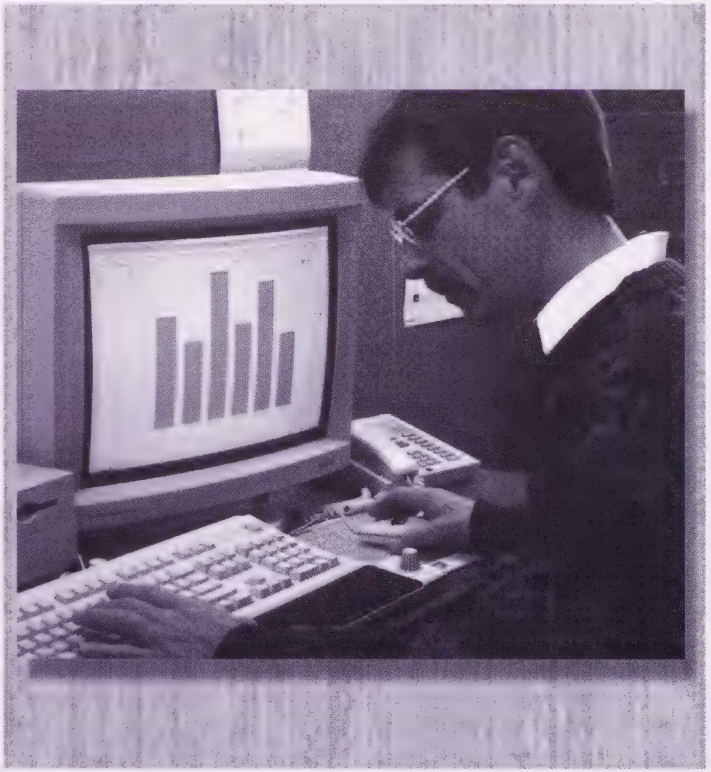
How many other situations can you think of that involve inequalities?

Assignment



You are now ready to complete the assignment for Section 2.

Module Summary



Were you able to discover the secret to the algebra problem posed in the Module Overview? Did you find that the unknown (chosen) number cancels out, thus making 7 the only possible solution, when the problem is translated into a mathematical statement and simplified?

In this module you examined and solved equations and inequations. You then applied your knowledge of equations and inequations to solving mathematical problems as well as real-world problems—translating word problems into mathematical statements and then solving.

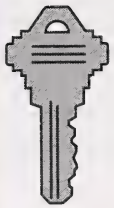
This module has given you the opportunity to expand your knowledge of algebra and has shown you how algebra can relate to some real-world situations. Algebra is an invaluable tool when it comes to understanding some real-world situations and solving some real-world problems. By successfully completing this module, your repertoire of problem-solving strategies has grown and become more powerful.

Final Module Assignment

Assignment Booklet

You are now ready to complete the final module assignment.

APPENDIX



- Glossary
- Suggested Answers
- Puzzles
- Cut-out Learning Aids

Glossary

Algebra: a branch of mathematics which describes basic arithmetic relations

Algebraic equation: an equation containing a variable

$2a = 6$ is an algebraic equation.

Algebraic expression: a combination of numerals, variables, and/or other mathematical symbols

$3a + 2$ is an algebraic expression.

Clearing the denominator: multiplying both sides of an equation by the lowest common multiple of the denominators to get the equation in a form where it has no fractions

Coefficient: a number multiplying a variable in a mathematical expression or equation

Cross products: the products that result from cross multiplying

$\frac{a}{b} \times \frac{c}{d}$ *ad* and *bc* are the cross products.

Formulas: general rules that use letters to represent various values

Inequation: a number sentence showing that the left side and the right side are not equal

$2m > 6$ is an inequation

Like terms: mathematical expressions that contain the same variable(s) with the same exponent

$7a^2$, $4a^2$, $-3a^2$, and $\frac{1}{2}a^2$ are like terms.

$5ab$, $4a^2b$, abc, and $15ab^2$ are not like terms.

Mathematic expression: the numbers and symbols used to represent an English phrase or sentence

Variable: a letter or symbol used to represent a number

Suggested Answers

Section 1: Activity 1

1. The equations in the video have like terms being added or subtracted.
2. Replacing each side of the equation by its inverse is the equivalent step in the model.
3. Modelling equations involving decimal numbers is inconvenient because it involves parts of a cylinder or disk.

4. a. $2x+4x=12$

$6x=12$

$\frac{6x}{6}=\frac{12}{6}$

$x=2$

Verification

LS	RS
$2x+4x$	12
$=2(2)+4(2)$	
$=4+8$	
$=12$	

LS = RS

b. $6y-2y=-8$

$4y=-8$

$\frac{4y}{4}=\frac{-8}{4}$

$y=-2$

Verification

LS	RS
$6y-2y$	-8
$=6(-2)-2(-2)$	
$=-12+4$	
$=-8$	

LS = RS

c. $a-3a=6$

$-2a=6$

$\frac{-2a}{-2}=\frac{6}{-2}$

$a=-3$

Verification

LS	RS
$a-3a$	6
$=-3-3(-3)$	
$=-3+9$	
$=6$	

LS = RS

d. $-3y+5y=15$

$2y=15$

$\frac{2y}{2}=\frac{15}{2}$

$y=7.5$

Verification

LS	RS
$-3y+5y$	15
$=-3(7.5)+5(7.5)$	
$=-22.5+37.5$	
$=15$	

LS = RS

e. $5a-0.5a=45$

$4.5a=45$

$\frac{4.5a}{4.5}=\frac{45}{4.5}$

$a=10$

Verification

LS	RS
$5a-0.5a$	45
$=5(10)-0.5(10)$	
$=50-5$	
$=45$	

LS = RS

f. $3.9a-3.2a=7$

$0.7a=7$

$\frac{0.7a}{0.7}=\frac{7}{0.7}$

$a=10$

Verification

LS	RS
$3.9a-3.2a$	7
$=3.9(10)-3.2(10)$	
$=39-32$	
$=7$	

LS = RS

5. The answer to the puzzle is **I LOVE MATH**.

6. Let n be the larger number. Thus, $n-18$ is the smaller number.

$$\begin{aligned} n+n-18 &= 100 \\ 2n-18 &= 100 \\ 2n-18+18 &= 100+18 \\ 2n &= 118 \\ \frac{2n}{2} &= \frac{118}{2} \\ n &= 59 \end{aligned}$$

Now find the smaller number.

$$\begin{aligned} n-18 &= 59-18 \\ &= 41 \end{aligned}$$

Verification

Reread the problem.

Is $41+59=100$? Yes.

Therefore, the numbers are 41 and 59.

7. Let b be the number of free throws made. Thus, $b+6$ is the number of free throws missed.

$$\begin{aligned} b+b+6 &= 70 \\ 2b+6 &= 70 \\ 2b+6-6 &= 70-6 \\ 2b &= 64 \\ \frac{2b}{2} &= \frac{64}{2} \\ b &= 32 \end{aligned}$$

Verification

Reread the problem.

Is six more than 32 equal to 38? Yes.
Is $32+38=70$? Yes.

Therefore, the basketball player made 32 free throws.

8. Let n be the number of pizzas Jason sold. Thus, $4n$ is the number of pizzas Ramona sold, and $n-5$ is the number of pizzas Alexi sold.

$$\begin{aligned} n+4n+n-5 &= 91 \\ 6n-5 &= 91 \\ 6n-5+5 &= 91+5 \\ 6n &= 96 \\ n &= 16 \end{aligned}$$

Now find the number of pizza sold by Ramona and by Alexi.

$4n = 4(16)$ $\qquad$ $n - 5 = 16 - 5$

$= 64$ $\qquad$ $= 11$

Verification

Reread the problem.

Is 4 times 16 equal to 64? Yes.
Is 5 less than 16 equal to 11? Yes.
Is $16 + 64 + 11 = 91$? Yes.

Therefore, Jason sold 16 pizzas, Ramona sold 64 pizzas, and Alexi sold 11 pizzas.

9. Let ℓ be the length of the shortest piece of pipe. Thus, $\ell + 17$ and 3ℓ are the lengths of the other two pieces.

$$\ell + \ell + 17 + 3\ell = 127$$
$$5\ell + 17 = 127$$
$$5\ell + 17 - 17 = 127 - 17$$
$$5\ell = 110$$
$$\frac{5\ell}{5} = \frac{110}{5}$$
$$\ell = 22$$

Now find the length of the other two pieces of pipe.

$\ell + 17 = 22 + 17$ $\qquad$ $3\ell = 3(22)$

$= 39$ $\qquad$ $= 66$

Verification

Reread the problem.

Is one piece 17 m longer than the shortest piece? Yes.
Is the third piece three times longer than the shortest piece? Yes.
Is the total length of the three pieces equal to 127 m? Yes.

Therefore, the three pieces of pipe are 22 m, 39 m, and 66 m in length.

10. **a.** The equation in Example 1 has brackets in it to indicate multiplication.

b. Yes, it is like those of $ax + b = c$.

11. **a.**
$$3(y - 2) = 6$$
$$3y - 6 = 6$$
$$3y - 6 + 6 = 6 + 6$$
$$3y = 12$$
$$\frac{3y}{3} = \frac{12}{3}$$
$$y = 4$$

Verification

LS	RS
$3(y-2)$	6
$= 3(4-2)$	
$= 3(2)$	
$= 6$	

LS = RS

b. $2(x+1)=-6$

$2x+2=-6$

$2x+2-2=-6-2$

$2x=-8$

$\frac{2x}{2}=\frac{-8}{2}$

$x=-4$

Verification

LS	RS
$2(x+1)$	-6
$=2(-4+1)$	
$=2(-3)$	
$=-6$	

LS = RS

c. $6(4+w)=18$

$24+6w=18$

$24+6w-24=18-24$

$6w=-6$

$\frac{6w}{6}=\frac{-6}{6}$

$w=-1$

Verification

LS	RS
$6(4+w)$	18
$=6[4+(-1)]$	
$=6(3)$	
$=18$	

LS = RS

d. $3(7-4x)=29$

$21-12x=29$

$21-12x-21=29-21$

$-12x=8$

$\frac{-12x}{-12}=\frac{8}{-12}$

$x=-\frac{2}{3}$

Verification

LS	RS
$3(7-4x)$	29
$=3\left[7-4\left(-\frac{2}{3}\right)\right]$	
$=3\left(\frac{21}{3}+\frac{8}{3}\right)$	
$=\cancel{3}^{1}\left(\frac{29}{\cancel{3}_{1}}\right)$	
$=29$	

LS = RS

12. Let n be the number.

$2(n-7)=25$

$2n-14=25$

$2n-14+14=25+14$

$2n=39$

$\frac{2n}{2}=\frac{39}{2}$

$n=19.5$

Verification

Is twice the difference between 19.5 and 7 equal to 25? Yes.

Therefore, the number is 19.5.

13. Let n be the amount Amir saved from his allowance. Thus, $n+5$ is the total he had after his father paid him.

$$\begin{aligned} 3(n+5) &= 27 \\ 3n+15 &= 27 \\ 3n+15-15 &= 27-15 \\ 3n &= 12 \\ \frac{3n}{3} &= \frac{12}{3} \\ n &= 4 \end{aligned}$$

Determine how much Amir made doing the odd jobs.

$$\begin{aligned} 27-(n+5) &= 27-(4+5) \\ &= 27-9 \\ &= 18 \end{aligned}$$

Verification

Is the amount required to buy the ticket triple the amount Amir had from his allowance plus what his father gave him? Yes.

Is the sum of what he had saved from his allowance, the amount his father gave him, and the amount he earned from his neighbour equal to \$27? Yes.

Therefore, Amir had saved \$4 from his allowance and he made \$18 doing the odd jobs for his neighbour.

14. Let x be the shorter lengths. Thus, $3x$ is the longer lengths.

$$\begin{aligned} 2(x)+2(3x) &= 200 \\ 2x+6x &= 200 \\ 8x &= 200 \\ \frac{8x}{8} &= \frac{200}{8} \\ x &= 25 \end{aligned}$$

Now find the longer length.

$$\begin{aligned} 3x &= 3(25) \\ &= 75 \end{aligned}$$

Verification

Reread the problem.

Is the sum of the lengths equal to 200 m? Yes.

Therefore, the two shorter pieces are 25 m each, and the two longer pieces are 75 m each.

15. Answers will vary. A sample problem is given.

Jerome, Sam, and Rodney each bought shirts that were the same price. Jerome bought 2 shirts, Sam bought 1 shirt, and Rodney bought 4 shirts. If the total the boys spent was \$168, what is the cost of one shirt?

Now Try This

16. Answers will vary. The following is one way of solving the problem.

If 1 chicken $(c)+2$ ducks $(d)+3$ geese $(g)=25$,
then $2c+4d+6g=50$, but $3c+1d=2g$.

Therefore, the $6g$ can be replaced by $9c+3d$.

$$\therefore\ 2c+4d+9c+3d = 50$$
$$11c+7d = 50$$

Since c, d, and g have to be whole numbers, the only replacements for c and d are $c=2$ and $d=4$.

$$\therefore\ 1c+2d+3g = 25$$
$$2+8+3g = 25$$
$$3g = 15$$
$$g = 5$$

Thus, ducks cost \$4, chickens cost \$2, and geese cost \$5.

17. Your completed spreadsheet should be similar to the following:

	A	B
1	Number Boys	18
2	Number Girls	25
3	Total	43

There are 18 boys and 25 girls in the class.

18. Your completed spreadsheet should be similar to the following:

	A	B	C
1	Loonies and Two-Dollar Coins		
2			
3		Number of Coins	Value
4	Loonies	75	\$ 75.00
5	Two-Dollar Coins	106	\$ 212.00
6	Total	181	\$ 287.00

There are 75 loonies and 106 two-dollar coins in the change machine.

19. Your completed spreadsheet should be similar to the following:

	A	B
1	Dimensions of a Picture	
2		
3	Width	60
4	Length	85
5	Perimeter	290

The largest picture Jodie can frame is 60 cm by 85 cm.

20. a. $A = P + I$

$= 1000 + 90$

$= 1090$

The amount paid to the lender is \$1090.

b. $A = P + I$

$1620 = P + 120$

$1620 - 120 = P + 120 - 120$

$1500 = P$

The principal is \$1500.

21. a. $d = 343t$

$= 343(6)$

$= 2058$

Fatima is 2058 m (or 2.058 km) from the storm.

b. $d = 343t$

$5000 = 343t$

$\frac{5000}{343} = \frac{343t}{343}$

$15 \doteq t$

5 km = 5000 m

It will be about 15 s between the time Randall sees the lightning and hears the thunder.

22. $A = \frac{1}{2}(b_1 + b_2) \times h$

$= \frac{1}{2}(11 + 18) \times 8$

$= \frac{1}{2}(29) \times 8$

$= 14.5 \times 8$

$= 116$

The area of the trapezoid is 116 cm^2.

23. a. Let c represent the cost (in dollars) and t represent the time (in minutes).

$\therefore\ c = 2.50 + 1.25(t - 3)$

b. $c = 2.50 + 1.25(t - 3)$

$= 2.50 + 1.25(10 - 3)$

$= 2.50 + 1.25(7)$

$= 2.50 + 8.75$

$= 11.25$

The cost of talking for 10 min is \$11.25.

c.

$$c = 2.50 + 1.25(t - 3)$$
$$20 = 2.50 + 1.25(t - 3)$$
$$20 = 2.50 + 1.25t - 3.75$$
$$20 = 1.25t - 1.25$$
$$20 + 1.25 = 1.25t - 1.25 + 1.25$$
$$21.25 = 1.25t$$
$$\frac{21.25}{1.25} = \frac{1.25t}{1.25}$$
$$17 = t$$

Yin can talk for 17 min.

24. The answer to the puzzle is **SHE PASSED THE BUCK**.

Now Try This

25. Set up an equation for Tony and for Yolanda. Calculate the number of days each would have to save in order to have $69.

Let d be the number of days.

Tony

$$29 + 3.20d = 69$$
$$29 + 3.20d - 29 = 69 - 29$$
$$3.20d = 40$$
$$\frac{3.20d}{3.20} = \frac{40}{3.20}$$
$$d = 12.5$$

Yolanda

$$20 + 3.50d = 69$$
$$20 + 3.50d - 20 = 69 - 20$$
$$3.50d = 49$$
$$\frac{3.50d}{3.50} = \frac{49}{3.50}$$
$$d = 14$$

Tony will need 13 days in order to save at least $69, and Yolanda will need 14 days in order to save $69.

Therefore, Tony will be able to buy the game first.

Section 1: Activity 2

1. a. -4 is the opposite of $+4$. By adding the opposite, it leaves $3m$ by itself on the left side of the equation.

b. $-m$ is the opposite of m. This operation removes the variable from the right side of the equation and collects them on the left side.

c. Both sides of the equation were divided by 2 in order to remove the 2 from in front of the variable (to isolate the variable).

2. a.

$$5n-8=3n$$
$$5n-8+8=3n+8$$
$$5n=3n+8$$
$$5n-3n=3n+8-3n$$
$$2n=8$$
$$\frac{2n}{2}=\frac{8}{2}$$
$$n=4$$

b.

$$10-d=-6d$$
$$10-d-10=-6d-10$$
$$-d=-6d-10$$
$$-d+6d=-6d-10+6d$$
$$5d=-10$$
$$\frac{5d}{5}=\frac{-10}{5}$$
$$d=-2$$

c.

$$4n+12=-2n$$
$$4n+12-12=-2n-12$$
$$4n=-2n-12$$
$$4n+2n=-2n-12+2n$$
$$6n=-12$$
$$\frac{6n}{6}=\frac{-12}{6}$$
$$n=-2$$

d.

$$3n+48=5n$$
$$3n+48-48=5n-48$$
$$3n=5n-48$$
$$3n-5n=5n-48-5n$$
$$-2n=-48$$
$$\frac{-2n}{-2}=\frac{-48}{-2}$$
$$n=24$$

e.

$$3.2a-14=1.2a$$
$$3.2a-14+14=1.2a+14$$
$$3.2a=1.2a+14$$
$$3.2a-1.2a=1.2a+14-1.2a$$
$$2a=14$$
$$\frac{2a}{2}=\frac{14}{2}$$
$$a=7$$

f.

$$1.6y=2.8y-6$$
$$1.6y-2.8y=2.8y-6-2.8y$$
$$-1.2y=-6$$
$$\frac{-1.2y}{-1.2}=\frac{-6}{-1.2}$$
$$y=5$$

3. a.

$$3x+9=2x-5$$
$$3x+9-9=2x-5-9$$
$$3x=2x-14$$
$$3x-2x=2x-14-2x$$
$$x=-14$$

b.

$$4-8a=2a-66$$
$$4-8a-4=2a-66-4$$
$$-8a=2a-70$$
$$-8a-2a=2a-70-2a$$
$$-10a=-70$$
$$\frac{-10a}{-10}=\frac{-70}{-10}$$
$$a=7$$

c.

$$3+2a=-5a-11$$
$$3+2a-3=-5a-11-3$$
$$2a=-5a-14$$
$$2a+5a=-5a-14+5a$$
$$7a=-14$$
$$\frac{7a}{7}=\frac{-14}{7}$$
$$a=-2$$

d.

$$8n-2=6n+8$$
$$8n-2+2=6n+8+2$$
$$8n=6n+10$$
$$8n-6n=6n+10-6n$$
$$2n=10$$
$$\frac{2n}{2}=\frac{10}{2}$$
$$n=5$$

e.

$$2.6c+7.6=1.2c-8.3$$
$$2.6c+7.6-7.6=1.2c-8.3-7.6$$
$$2.6c=1.2c-15.9$$
$$2.6c-1.2c=1.2c-15.9-1.2c$$
$$1.4c=-15.9$$
$$\frac{1.4c}{1.4}=\frac{-15.9}{1.4}$$
$$c=-11.4 \text{ (rounded to one decimal place)}$$

f.

$$1.1+3.2s=5s+4.3$$
$$1.1+3.2s-1.1=5s+4.3-1.1$$
$$3.2s=5s+3.2$$
$$3.2s-5s=5s+3.2-5s$$
$$-1.8s=3.2$$
$$\frac{-1.8s}{-1.8}=\frac{3.2}{-1.8}$$
$$s=-1.8 \text{ (rounded to one decimal place)}$$

4. The following steps describe how to model the solution to the equation $3x - 2 = x$.

Step 1: Model the equation by placing 3 positive cylinders and 2 negative counters on the left side of the balance and 1 positive cylinder on the right side.

Step 2: To isolate $3x$ on the left, place 2 positive counters on each side of the balance.

Step 3: Simplify by removing the zeros from the left side.

Step 4: To isolate the constant (2) on the right, place a negative cylinder on each side.

Step 5: Simplify by removing the zeros from each side.

Step 6: To isolate x, divide each side into two groups. The solution is the value of one group.

5. Let n be Anton's hourly wage. Thus, $3n$ is Mr. Mironov's hourly wage.

$$\begin{aligned} n + 11.60 &= 3n \\ n + 11.60 - 11.60 &= 3n - 11.60 \\ n &= 3n - 11.60 \\ n - 3n &= 3n - 11.60 - 3n \\ -2n &= -11.60 \\ \frac{-2n}{-2} &= \frac{-11.60}{-2} \\ n &= 5.80 \end{aligned}$$

Now find Mr. Mironov's hourly wage.

$$\begin{aligned} 3n &= 3 \times 5.80 \\ &= 17.40 \end{aligned}$$

Anton makes \$5.80/h and his father makes \$17.40/h.

6. Since the cost of each pizza is to be the same, set the two equations equal to each other and solve for t.

$$\begin{aligned} 5.90 + 0.90t &= 7.25 + 0.45t \\ 5.90 + 0.90t - 5.90 &= 7.25 + 0.45t - 5.90 \\ 0.90t &= 1.35 + 0.45t \\ 0.90t - 0.45t &= 1.35 + 0.45t - 0.45t \\ 0.45t &= 1.35 \\ \frac{0.45t}{0.45} &= \frac{1.35}{0.45} \\ t &= 3 \end{aligned}$$

The cost is the same when 3 toppings are ordered.

7. The answer to the puzzle is as follows:

Tragedy on the Cliff by **EILEEN DOVER**
Mystery of the Creaking Door by **RUSTY HINGES**
P.S. by **ADALINE MOORE**

8. Answers will vary. One possible scenario is given.

Lisa works at Wholesale Warehouse. Her supervisor makes four times as much as she makes. If Lisa made \$17.40/h more, then she would make the same wage as her supervisor. Find Lisa's and her supervisor's hourly wage.

9. a.

$$\begin{aligned} 5(3x-8) &= 2(2x+13) \\ 15x-40 &= 4x+26 \\ 15x-40+40 &= 4x+26+40 \\ 15x &= 4x+66 \\ 15x-4x &= 4x+66-4x \\ 11x &= 66 \\ \frac{11x}{11} &= \frac{66}{11} \\ x &= 6 \end{aligned}$$

Verification

LS	RS
$5(3x-8)$	$2(2x+13)$
$=5[3(6)-8]$	$=2[2(6)+13]$
$=5(18-8)$	$=2(12+13)$
$=5(10)$	$=2(25)$
$=50$	$=50$

LS = RS

Therefore, the solution is $x=6$.

b.

$$\begin{aligned} 4(a+6) &= 8(a+4) \\ 4a+24 &= 8a+32 \\ 4a+24-24 &= 8a+32-24 \\ 4a &= 8a+8 \\ 4a-8a &= 8a+8-8a \\ -4a &= 8 \\ \frac{-4a}{-4} &= \frac{8}{-4} \\ a &= -2 \end{aligned}$$

Verification

LS	RS
$4(a+6)$	$8(a+4)$
$=4(-2+6)$	$=8(-2+4)$
$=4(4)$	$=8(2)$
$=16$	$=16$

LS = RS

Therefore, the solution is $a=-2$.

c. $-4(2x-3)=2(-x+3)$

$$-8x+12=-2x+6$$
$$-8x+12-12=-2x+6-12$$
$$-8x=-2x-6$$
$$-8x+2x=-2x-6+2x$$
$$-6x=-6$$
$$\frac{-6x}{-6}=\frac{-6}{-6}$$
$$x=1$$

Verification

LS	RS
$-4(2x-3)$	$2(-x+3)$
$=-4[2(1)-3]$	$=2[-(1)+3]$
$=-4(2-3)$	$=2(-1+3)$
$=-4(-1)$	$=2(2)$
$=4$	$=4$

LS = RS

Therefore, the solution is $x=1$.

d. $0.6(2h+6)=1.4(h+2)$

$$1.2h+3.6=1.4h+2.8$$
$$1.2h+3.6-3.6=1.4h+2.8-3.6$$
$$1.2h=1.4h-0.8$$
$$1.2h-1.4h=1.4h-0.8-1.4h$$
$$-0.2h=-0.8$$
$$\frac{-0.2h}{-0.2}=\frac{-0.8}{-0.2}$$
$$h=4$$

Verification

LS	RS
$0.6(2h+6)$	$1.4(h+2)$
$=0.6[2(4)+6]$	$=1.4(4+2)$
$=0.6(8+6)$	$=1.4(6)$
$=0.6(14)$	$=8.4$
$=8.4$	

LS = RS

Therefore, the solution is $h=4$.

10. Let x be Uncle Julius' present age.

$$\begin{aligned} x+9 &= 2(x-20) \\ x+9 &= 2x-40 \\ x+9-9 &= 2x-40-9 \\ x &= 2x-49 \\ x-2x &= 2x-49-2x \\ -x &= -49 \\ x &= 49 \end{aligned}$$

Verification

Reread the problem.

How old will he be in 9 years? $49+9=58$.
How old was he 20 years ago? $49-20=29$.
Is 58 twice 29? Yes.

Therefore, Freda's Uncle Julius is 49 years old.

11. Let x be the width of the enclosure. Thus, $2x+4$ is the length.

$$\begin{aligned} 2(x)+2(2x+4) &= 56 \\ 2x+4x+8 &= 56 \\ 6x+8 &= 56 \\ 6x+8-8 &= 56-8 \\ 6x &= 48 \\ \frac{6x}{6} &= \frac{48}{6} \\ x &= 8 \end{aligned}$$

Now find the length of the enclosure.

$$\begin{aligned} 2x+4 &= 2(8)+4 \\ &= 16+4 \\ &= 20 \end{aligned}$$

Verification

Reread the problem.

Is the length 4 m more than twice the width? Yes.
Is the perimeter equal to 56 m? Yes.

Therefore, the width is 8 m and the length is 20 m.

12.

People	Present Age	Age Two Years Ago
Vladimir	$2(n+4)$	$2(n+4)-2$
Enzo	n	$n-2$

Write and solve the equation.

$$\begin{aligned}
2(n+4)-2 &= 3(n-2)\\
2n+8-2 &= 3n-6\\
2n+6 &= 3n-6\\
2n+6-6 &= 3n-6-6\\
2n &= 3n-12\\
2n-3n &= 3n-12-3n\\
-n &= -12\\
n &= 12
\end{aligned}$$

Now find Vladimir's present age.

$$\begin{aligned}
2(n+4) &= 2(12+4)\\
&= 2(16)\\
&= 32
\end{aligned}$$

Enzo is presently 12 years old, and Vladimir is presently 32 years old.

13. a. Let w be Mrs. Waslyk's investment. Thus, $5w$ is Mrs. Jackson's investment, and $3(w-20)$ is Mrs. Winkler's investment.

Write and solve the equation.

$$\begin{aligned}
5w-3(w-20) &= w+1000\\
5w-3w+60 &= w+1000\\
2w+60 &= w+1000\\
2w+60-60 &= w+1000-60\\
2w &= w+940\\
2w-w &= w+940-w\\
w &= 940
\end{aligned}$$

Now determine Mrs. Jackson's and Mrs. Winkler's investment.

$$\begin{aligned}
5w &= 5(940)\\
&= 4700
\end{aligned}$$

$$\begin{aligned}
3(w-20) &= 3(940-20)\\
&= 3(920)\\
&= 2760
\end{aligned}$$

Therefore, Mrs. Waslyk's initial investment was \$940, Mrs. Jackson's was \$4700, and Mrs. Winkler's was \$2760.

b. $940 \div 400 = 2.35$

Therefore, each share cost \$2.35.

c. $\dfrac{1300-940}{400} = \dfrac{360}{400}$

$= 0.9$

Each share has increased in price by \$0.90.

d. Percent increase $= \dfrac{0.90}{2.35} \times 100$

$\doteq 0.3830 \times 100$

$\doteq 38\%$

The percent increase in the value of Mrs. Waslyk's shares is approximately 38%.

14.

Type of Coin	Number of Coins	Value of Coin (¢)	Total Value (¢)
nickel	n	5	$5n$
dime	$2n$	10	$10(2n) = 20n$
quarter	$4(2n) = 8n$	25	$25(8n) = 200n$
loonie	$8n+10$	100	$100(8n+10)$ $= 800n+1000$

Write and solve the equation.

$$\begin{aligned} 5n + 20n + 200n + 800n + 1000 &= 13\,300 \\ 1025n + 1000 &= 13\,300 \\ 1025n + 1000 - 1000 &= 13\,300 - 1000 \\ 1025n &= 12\,300 \\ \frac{1025n}{1025} &= \frac{12\,300}{1025} \\ n &= 12 \end{aligned}$$

Now determine the number of dimes, quarters, and loonies individually.

$2n = 2(12)$ $\quad$ $8n = 8(12)$ $\quad$ $8n+10 = 8(12)+10$

$= 24$ $\quad$ $= 96$ $\quad$ $= 96+10$

$= 106$

Therefore, there were 12 nickels, 24 dimes, 96 quarters, and 106 loonies in the vending machine.

15. The answer to the puzzle is **A LUNA TICK**.

Now Try This

16. If the length and width of the photograph is doubled, the area will be four times the original; thus, the cost of the enlargement will be four times the cost of the original.

Section 1: Activity 3

1. The coefficient of $3a$ is 3, and the coefficient of $-2a$ is -2.

2. a. $\frac{n}{3}$ means $\frac{1}{3}\times n$.

b. The coefficient of $\frac{n}{3}$ is $\frac{1}{3}$.

3. The coefficient of $\frac{4}{m}$ is 4 since $\frac{4}{m}$ means $4\times\frac{1}{m}$.

4. The coefficient of $\frac{n+4}{6}$ is $\frac{1}{6}$ since $\frac{n+4}{6}$ means $\frac{1}{6}(n+4)$.

5. a. 15 **b.** 9 **c.** 5 **d.** 20

6. a. $\frac{n}{15}=\frac{2}{3}$

$\frac{n}{\cancel{15}_{1}}\times\frac{\overset{1}{\cancel{15}}}{1}=\frac{2}{\cancel{3}_{1}}\times\frac{\overset{5}{\cancel{15}}}{1}$

$n=10$

Verification

LS	RS
$\frac{n}{15}$	$\frac{2}{3}$
$=\frac{10}{15}$	
$=\frac{2}{3}$	

LS = RS

b. $\frac{f}{9}=\frac{8}{3}$

$\frac{f}{\cancel{9}_{1}}\times\frac{\overset{1}{\cancel{9}}}{1}=\frac{8}{\cancel{3}_{1}}\times\frac{\overset{3}{\cancel{9}}}{1}$

$f=24$

Verification

LS	RS
$\frac{f}{9}$	$\frac{8}{3}$
$=\frac{24}{9}$	
$=\frac{8}{3}$	

LS = RS

c. $\frac{y}{5}=\frac{5}{6}$

$\frac{y}{\cancel{5}_{1}}\times\frac{\overset{1}{\cancel{5}}}{1}=\frac{5}{6}\times 5$

$y=\frac{25}{6}$ or $4\frac{1}{6}$

Verification

LS	RS
$\frac{y}{5}$	$\frac{5}{6}$
$=\frac{\frac{25}{6}}{5}$	
$=\frac{\frac{\overset{5}{\cancel{25}}}{6}\times\frac{1}{\cancel{5}_{1}}}{\frac{1}{\cancel{5}_{1}}\times\frac{1}{\cancel{5}_{1}}}$	
$=\frac{5}{6}$	

LS = RS

d. $\frac{3}{4} = \frac{c}{20}$

$\frac{3}{\cancel{4}_{1}} \times \cancel{20}^{5} = \frac{c}{\cancel{20}_{1}} \times \cancel{20}^{1}$

$15 = c$

Verification

LS	RS
$\frac{3}{4}$	$\frac{c}{20}$
	$= \frac{15}{20}$
	$= \frac{3}{4}$

LS = RS

7. a. $\frac{n}{5} = \frac{3}{4}$

$\frac{n}{\cancel{5}_{1}} \times \cancel{20}^{4} = \frac{3}{\cancel{4}_{1}} \times \cancel{20}^{5}$

$4n = 15$

$\frac{4n}{4} = \frac{15}{4}$

$n = \frac{15}{4}$ or $3\frac{3}{4}$

Verification

LS	RS
$\frac{n}{5}$	$\frac{3}{4}$
$= \frac{\frac{15}{4}}{5}$	
$= \frac{\frac{\cancel{15}^{3}}{4} \times \frac{1}{\cancel{5}_{1}}}{\frac{1}{\cancel{5}_{1}} \times \frac{1}{\cancel{5}_{1}}}$	
$= \frac{3}{4}$	

LS = RS

b. $\frac{2}{n} = \frac{5}{8}$

$\frac{2}{\cancel{n}_{1}} \times 8\cancel{n}^{1} = \frac{5}{\cancel{8}_{1}} \times \cancel{8}^{1}n$

$16 = 5n$

$5n = 16$

$\frac{5n}{5} = \frac{16}{5}$

$n = \frac{16}{5}$ or $3\frac{1}{5}$

Verification

LS	RS
$\frac{2}{n}$	$\frac{5}{8}$
$= \frac{2}{\frac{16}{5}}$	
$= \frac{\frac{\cancel{2}^{1}}{1} \times \frac{5}{\cancel{16}^{8}}}{\frac{1}{\cancel{16}_{\cancel{5}_{1}}} \times \frac{1}{\cancel{5}_{1}}}$	
$= \frac{5}{8}$	

LS = RS

c. $\frac{x}{3} = \frac{4}{5}$

$\frac{x}{\cancel{3}_{1}} \times \cancel{15}^{5} = \frac{4}{\cancel{5}_{1}} \times \cancel{15}^{3}$

$5x = 12$

$x = \frac{12}{5}$ or $2\frac{2}{5}$

Verification

LS	RS
$\frac{x}{3}$	$\frac{4}{5}$
$= \frac{\frac{12}{5}}{3}$	
$= \frac{\frac{\cancel{12}^{4}}{5} \times \frac{1}{\cancel{3}_{1}}}{\frac{1}{\cancel{3}_{1}} \times \frac{1}{\cancel{3}_{1}}}$	
$= \frac{4}{5}$	

LS = RS

d. $\frac{3}{r} = \frac{5}{8}$

$\frac{\cancel{3}^{1}}{\cancel{r}_{1}} \times \cancel{8}^{1}r = \frac{5}{\cancel{8}_{1}} \times \cancel{8}^{1}r$

$24 = 5r$

$5r = 24$

$\frac{5r}{5} = \frac{24}{5}$

$r = \frac{24}{5}$ or $4\frac{4}{5}$

Verification

LS	RS
$\frac{3}{r}$	$\frac{5}{8}$
$= \frac{3}{\frac{24}{5}}$	
$= \frac{\frac{1}{\cancel{3}_{1}} \times \frac{5}{\cancel{24}_{8}}}{\frac{1}{\cancel{24}_{1}} \times \frac{1}{\cancel{5}_{1}}}$	
$= \frac{5}{8}$	

LS = RS

8. a. $\frac{n}{5} \times \frac{3}{4}$

$n \times 4 = 3 \times 5$

$4n = 15$

$\frac{4n}{4} = \frac{15}{4}$

$n = \frac{15}{4}$ or $3\frac{3}{4}$

b. $\frac{2}{n} \times \frac{5}{8}$

$2 \times 8 = 5 \times n$

$16 = 5n$

$5n = 16$

$\frac{5n}{5} = \frac{16}{5}$

$n = \frac{16}{5}$ or $3\frac{1}{5}$

c. $\frac{x}{3} \times \frac{4}{5}$

$x \times 5 = 4 \times 3$

$5x = 12$

$\frac{5x}{5} = \frac{12}{5}$

$x = \frac{12}{5}$ or $2\frac{2}{5}$

d. $\frac{3}{r} \times \frac{5}{8}$

$3 \times 8 = 5 \times r$

$24 = 5r$

$5r = 24$

$\frac{5r}{5} = \frac{24}{5}$

$r = \frac{24}{5}$ or $4\frac{4}{5}$

9. a. Step 1: Multiply both sides of the equation by the lowest common denominator. This is 20.

Step 2: Simplify both sides of the equation.

Step 3: Divide both sides of the equation by the factor that is multiplying the unknown.

Step 4: Simplify so you have the unknown only on one side and a number value on the other side.

b. Step 1: Cross-multiply a factor in the numerator with a factor in the denominator.

Step 2: Write the products of the factors.

Step 3: Divide each side of the equation by the factor that is multiplying the unknown.

Step 4: Simplify so you have the unknown only on one side and a number value on the other side.

10. **a.** 3 × 5 ÷ 4 =

3.75

b. 2 × 8 ÷ 5 =

3.2

c. 4 × 3 ÷ 5 =

2.4

d. 3 × 8 ÷ 5 =

4.8

11. Questions 11.a. and 11.b. are solved using cross products.

a. Let c be the number of containers Marvin can pick in 8 h.

$$\frac{32}{5} = \frac{c}{8}$$

$$\frac{32}{5} \times \frac{c}{8}$$

$$32 \times 8 = c \times 5$$

$$256 = 5c$$

$$5c = 256$$

$$\frac{5c}{5} = \frac{256}{5}$$

$$c = 51.2$$

Verification

Reread the question.

Should Marvin be able to pick more or less than 32 containers in 8 h? Yes, he should be able to pick more than 32 containers in 8 h.

Therefore, Marvin can pick 51.2 (51 full) containers in 8 h.

b. Let h be the number of hours needed to pick 112 containers of blueberries.

$$\frac{32}{5} = \frac{112}{h}$$

$$\frac{32}{5} \times \frac{112}{h}$$

$$32 \times h = 112 \times 5$$

$$32h = 560$$

$$\frac{32h}{32} = \frac{560}{32}$$

$$h = 17.5$$

Verification

Reread the problem.

17.5 h is more than 3 times 5 h.

Is 112 containers more than 3 times 32 containers? Yes.

Therefore, it will take Marvin 17.5 h to pick 112 containers of wild blueberries.

12. Let n be the smaller part. Thus, $n+4$ is the larger part.

$$\frac{n}{n+4} = \frac{5}{7}$$

$$\frac{n}{n+4} \times \frac{5}{7}$$

$$n \times 7 = 5(n+4)$$

$$7n = 5n + 20$$

$$7n - 5n = 5n + 20 - 5n$$

$$2n = 20$$

$$\frac{2n}{2} = \frac{20}{2}$$

$$n = 10$$

Now find the larger part.

$$n + 4 = 10 + 4$$
$$= 14$$

Verification

Reread the problem.

Is the sum of 10 and 14 equal to 24? Yes.

Do the numbers 10 and 14 have a ratio of 5 to 7? Yes.

Therefore, the smaller part is 10 and the larger part is 14.

13. a. $t = \frac{n}{8} + 5$

$= \frac{80}{8} + 5$

$= 10 + 5$

$= 15$

The temperature is about 15°C.

b. $t = \frac{n}{8} + 5$

$25 = \frac{n}{8} + 5$

$25 - 5 = \frac{n}{8} + 5 - 5$

$20 = \frac{n}{8}$

$20 \times 8 = \frac{n}{\cancel{8}_{1}} \times \overset{1}{\cancel{8}}$

$160 = n$

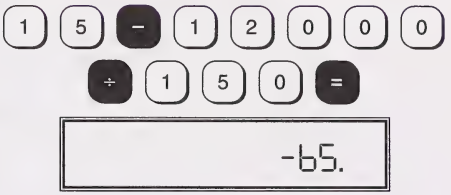
$n = 160$

At 25°C, a cricket should chirp about 160 times in one minute.

14. a. Altitude of 12 000 m

$T = t - \frac{h}{150}$

$T = 15 - \frac{12\,000}{150}$

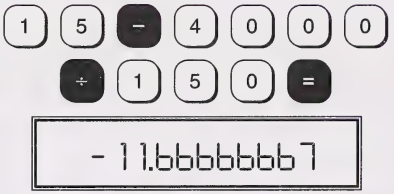


The temperature outside an aircraft at an altitude of 12 000 m would be $-65°C$.

Altitude of 4000 m

$T = t - \frac{h}{150}$

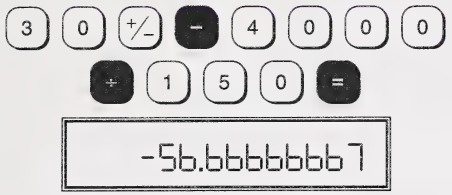
$T = 15 - \frac{4000}{150}$



The temperature outside an aircraft at an altitude of 4000 m would be approximately $-12°C$.

b. $T = t - \frac{h}{150}$

$T = -30 - \frac{4000}{150}$



The temperature outside the aircraft is approximately $-57°\text{C}$.

15. a. $C = \pi d$

$\frac{C}{\pi} = \frac{\pi d}{\pi}$

$\frac{C}{\pi} = d$

$d = \frac{C}{\pi}$

b. $P = a + b + c$

$P - b = a + b + c - b$

$P - b = a + c$

$P - b - c = a + c - c$

$P - b - c = a$

$a = P - b - c$

c. $A = 2b + c$

$A - 2b = 2b + c - 2b$

$A - 2b = c$

$c = A - 2b$

d. $A = \frac{1}{2}ab$

$A \times 2 = \frac{1}{\cancel{2}}ab \times \overset{1}{\cancel{2}}$

$2A = ab$

$\frac{2A}{a} = \frac{ab}{a}$

$\frac{2A}{a} = b$

$b = \frac{2A}{a}$

e. $I = Prt$

$\frac{I}{rt} = \frac{Prt}{rt}$

$\frac{I}{rt} = P$

$P = \frac{I}{rt}$

f. $V = \ell wh$

$\frac{V}{wh} = \frac{\ell wh}{wh}$

$\frac{V}{wh} = \ell$

$\ell = \frac{V}{wh}$

g.

$$A = \frac{1}{2}(b_1 + b_2) \times h$$

$$A \times 2 = \frac{1}{\cancel{2}}(b_1 + b_2) \times h \times \cancel{2}^{1}$$

$$2A = (b_1 + b_2) \times h$$

$$\frac{2A}{(b_1 + b_2)} = \frac{\cancel{(b_1 + b_2)} \times h}{\cancel{(b_1 + b_2)}}$$

$$\frac{2A}{(b_1 + b_2)} = h$$

$$h = \frac{2A}{(b_1 + b_2)}$$

16. a.

$$d = \frac{C}{\pi}$$

$$= \frac{27.004}{\pi}$$

$$\doteq 8.6 \text{ cm}$$

b.

$$a = P - b - c$$

$$= 48.5 - 7.6 - 19.4$$

$$= 21.5 \text{ cm}$$

c.

$$b = \frac{2A}{a}$$

$$= \frac{2(63.6)}{10.6}$$

$$= \frac{127.2}{10.6}$$

$$= 12 \text{ cm}$$

d.

$$\ell = \frac{V}{wh}$$

$$= \frac{336}{8.4(3.2)}$$

$$= \frac{336}{26.88}$$

$$= 12.5 \text{ cm}$$

e.

$$h = \frac{2A}{(b_1 + b_2)}$$

$$= \frac{2(126)}{12 + 18}$$

$$= \frac{252}{30}$$

$$= 8.4 \text{ cm}$$

17. Both formulas contain the same variables. In the first formula you are solving for distance in terms of velocity and time. In the second formula you are solving for velocity in terms of distance and time.

18. This question can be solved in two ways.

Method 1

Solve for n; then substitute the known value.

$$c = 50 + 0.1n$$

$$c - 50 = 50 + 0.1n - 50$$

$$c - 50 = 0.1n$$

$$\frac{c - 50}{0.1} = \frac{0.1n}{0.1}$$

$$\frac{c - 50}{0.1} = n$$

$$n = \frac{c - 50}{0.1}$$

If $c = 350$, then $n = \dfrac{c - 50}{0.1}$

$$= \frac{350 - 50}{0.1}$$

$$= \frac{300}{0.1}$$

$$= 3000$$

There were 3000 tickets printed.

Method 2

Substitute the known value; then solve for n.

If $c = 350$, then $c = 50 + 0.1n$

$$350 = 50 + 0.1n$$

$$350 - 50 = 50 + 0.1n - 50$$

$$300 = 0.1n$$

$$\frac{300}{0.1} = \frac{0.1n}{0.1}$$

$$3000 = n$$

There were 3000 tickets printed.

Now Try This

19. a. $150 \times \dfrac{4}{5} = 120$

There are 120 new vehicles in the lot.

$120 \times \dfrac{3}{4} = 90$

There are 90 new cars in the lot.

$90 \times \dfrac{2}{3} = 60$

There are 60 new, American-made cars in the lot.

$60 \times \dfrac{1}{2} = 30$

There are 30 new, American-made cars with air conditioning in the lot.

b. $\dfrac{\overset{1}{\cancel{4}}}{5} \times \dfrac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \dfrac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \dfrac{1}{\underset{1}{\cancel{2}}} = \dfrac{1}{5}$

$\dfrac{1}{5} \times 150 = 30$

There are 30 new, American-made cars with air conditioning in the lot.

c. You could work in reverse by multiplying by the reciprocal.

$$24 \times \frac{2}{1} = 48$$

There are 48 American-made cars.

$$48 \times \frac{3}{2} = 72$$

There are 72 cars.

$$72 \times \frac{4}{3} = 96$$

There are 96 new vehicle.

Note: You could also multiply 24 by 5 (the reciprocal of $\frac{1}{5}$) to get the total of 120 and then find $\frac{4}{5}$ of 120.

Section 1: Follow-up Activities

Extra Help

Note: Any method can be used to solve these problems. For these problems, the method of solving with equations is shown.

1. Let the number of quarters be n. Thus, the number of dimes is $2n$.

$$\begin{aligned} 25n + 10(2n) &= 900 \\ 25n + 20n &= 900 \\ 45n &= 900 \\ \frac{45n}{45} &= \frac{900}{45} \\ n &= 20 \end{aligned}$$

Now find the number of dimes.

$$\begin{aligned} 2n &= 2 \times 20 \\ &= 40 \end{aligned}$$

Thus, Lori has 40 dimes.

2.

	Present Age	Age in Eight Years
Mrs. Chan	c	$c+8$
Mrs. Wong	$c+9$	$c+17$

$$\begin{aligned} 7(c+8) &= 6(c+17) \\ 7c + 56 &= 6c + 102 \\ 7c + 56 - 56 &= 6c + 102 - 56 \\ 7c &= 6c + 46 \\ 7c - 6c &= 6c + 46 - 6c \\ c &= 46 \end{aligned}$$

Now find Mrs. Wong's age.

$$\begin{aligned} c+9 &= 46+9 \\ &= 55 \end{aligned}$$

Mrs. Chan is presently 46 years old, and Mrs. Wong is presently 55 years old.

3. Let the price of the running shoes be r. Thus, the price of the track shoes is $2r$.

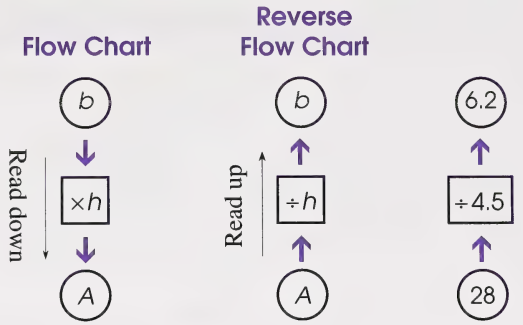
$$\begin{aligned} 2(2r)+2r &= 330 \\ 4r+2r &= 330 \\ 6r &= 330 \\ r &= 55 \end{aligned}$$

Now find the price of the track shoes.

$$\begin{aligned} 2r &= 2(55) \\ &= 110 \end{aligned}$$

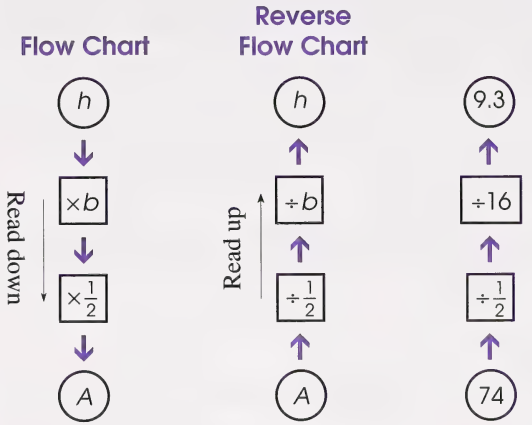
The running shoes cost \$55, and the track shoes cost \$110.

4. a.



The value of b is 6.2 cm, rounded to one decimal place.

b.



The value of h is 9.3, rounded to one decimal place.

Enrichment

1. a. Clearing the Denominator

$$\frac{3m}{9}=\frac{7}{6}$$

$$\frac{3m}{\cancel{9}_{1}}\times\frac{\cancel{18}^{2}}{1}=\frac{7}{\cancel{6}_{1}}\times\frac{\cancel{18}^{3}}{1}$$

$$6m=21$$

$$\frac{6m}{6}=\frac{21}{6}$$

$$m=3.5$$

Using Cross Products

$$\frac{3m}{9}\times\frac{7}{6}$$

$$3m\times 6=7\times 9$$

$$18m=63$$

$$\frac{18m}{18}=\frac{63}{18}$$

$$m=3.5$$

b. Clearing the Denominator

$$\frac{y+2}{5}=\frac{2y+1}{9}$$

$$\frac{y+2}{\cancel{5}_{1}}\times\frac{\cancel{45}^{9}}{1}=\frac{2y+1}{\cancel{9}_{1}}\times\frac{\cancel{45}^{5}}{1}$$

$$9(y+2)=5(2y+1)$$

$$9y+18=10y+5$$

$$9y+18-18=10y+5-18$$

$$9y=10y-13$$

$$9y-10y=10y-13-10y$$

$$-y=-13$$

$$y=13$$

Using Cross Products

$$\frac{y+2}{5}\times\frac{2y+1}{9}$$

$$9y+18=10y+5$$

$$9y+18-18=10y+5-18$$

$$9y=10y-13$$

$$9y-10y=10y-13-10y$$

$$-y=-13$$

$$y=13$$

c. Clearing the Denominators

$$\frac{2d}{5}+\frac{d}{10}=\frac{15}{2}$$

$$10\left(\frac{2d}{5}+\frac{d}{10}\right)=\frac{15}{2}\times 10$$

$$\cancel{10}^{2}\left(\frac{2d}{\cancel{5}_{1}}\right)+\cancel{10}^{1}\left(\frac{d}{\cancel{10}_{1}}\right)=\frac{15}{\cancel{2}_{1}}\times\frac{\cancel{10}^{5}}{1}$$

$$4d+d=75$$

$$5d=75$$

$$d=15$$

You cannot use the cross-product method directly here.

d. Clearing the Denominators

$$\frac{2n}{5}=\frac{n-6}{4}$$

$$\frac{2n}{\cancel{5}_{1}}\times\cancel{20}^{4}=\frac{n-6}{\cancel{4}_{1}}\times\cancel{20}^{5}$$

$$4(2n)=5(n-6)$$

$$8n=5n-30$$

$$8n-5=5n-30-5n$$

$$\frac{3n}{3}=\frac{-30}{3}$$

$$n=-10$$

Using Cross Products

$$\frac{2n}{5}\times\frac{n-6}{4}$$

$$4(2n)=5(n-6)$$

$$8n=5n-30$$

$$8n-5n=5n-30-5n$$

$$3n=-30$$

$$\frac{3n}{3}=\frac{-30}{3}$$

$$n=-10$$

e. Clearing the Denominators

$$\frac{n}{2}+\frac{n}{3}=n-\frac{1}{8}$$

$$24\left(\frac{n}{2}+\frac{n}{3}\right)=24\left(n-\frac{1}{8}\right)$$

$$\cancel{24}^{12}\left(\frac{n}{\cancel{2}_{1}}\right)+\cancel{24}^{8}\left(\frac{n}{\cancel{3}_{1}}\right)=24n-\cancel{24}^{3}\left(\frac{1}{\cancel{8}_{1}}\right)$$

$$12n+8n=24n-3$$

$$20n=24n-3$$

$$20n-24n=24n-3-24n$$

$$-4n=-3$$

$$\frac{-4n}{-4}=\frac{-3}{-4}$$

$$n=\frac{3}{4}$$

f. Clearing the Denominators

$$\frac{5}{n+3}=\frac{9}{2n}$$

$$2n\cancel{(n+3)}^{1}\left(\frac{5}{\cancel{n+3}_{1}}\right)=\cancel{2n}^{1}(n+3)\left(\frac{9}{\cancel{2n}_{1}}\right)$$

$$10n=9n+27$$

$$10n-9n=9n+27-9n$$

$$n=27$$

Using Cross Products

$$\frac{5}{n+3} \times \frac{9}{2n}$$

$$5(2n) = 9(n+3)$$
$$10n = 9n+27$$
$$10n-9n = 9n+27-9n$$
$$n = 27$$

g. ### Clearing the Denominators

$$\frac{a}{2}+\frac{a}{3}+\frac{a}{4}+\frac{a}{6}=15$$

$$\cancel{12}^{6}\left(\frac{a}{\cancel{2}_{1}}\right)+\cancel{12}^{4}\left(\frac{a}{\cancel{3}_{1}}\right)+\cancel{12}^{3}\left(\frac{a}{\cancel{4}_{1}}\right)+\cancel{12}^{2}\left(\frac{a}{\cancel{6}_{1}}\right)=15\times 12$$

$$6a+4a+3a+2a=180$$
$$15a=180$$
$$\frac{15a}{15}=\frac{180}{15}$$
$$a=12$$

h. ### Clearing the Denominators

$$\frac{5}{2(n+3)}=\frac{3}{n+4}$$

$$\cancel{2}^{1}\left(\cancel{n+3}_{1}\right)(n+4)\frac{5}{\cancel{2}_{1}\left(\cancel{n+3}_{1}\right)}=2(n+3)\left(\cancel{n+4}^{1}\right)\frac{3}{\cancel{n+4}_{1}}$$

$$5n+20=6n+18$$
$$5n+20-20=6n+18-20$$
$$5n=6n-2$$
$$5n-6n=6n-2-6n$$
$$-n=-2$$
$$n=2$$

Using Cross Products

$$\frac{5}{2(n+3)} \times \frac{3}{n+4}$$

$$5(n+4) = 3(2)(n+3)$$
$$5n+20 = 6n+18$$
$$5n+20-20 = 6n+18-20$$
$$5n = 6n-2$$
$$5n-6n = 6n-2-6n$$
$$-n = -2$$
$$n = 2$$

2. a. Let n be the total value of Mr. Johnson's estate. Thus, Arlene's share is $\frac{n}{2}$, Brad's share is $\frac{n}{4}$, and Jill's share is $\frac{n}{5}$.

The three charities share \$10 200.

Write the equation.

$$\frac{n}{2}+\frac{n}{4}+\frac{n}{5}+10\,200=n$$

Multiply both sides of the equation by 20 (the LCM of 2, 4, and 5).

$$20\left(\frac{n}{2}+\frac{n}{4}+\frac{n}{5}+10\,200\right)=n\times 20$$

$$\overset{10}{\cancel{20}}\left(\frac{n}{\cancel{2}_{1}}\right)+\overset{5}{\cancel{20}}\left(\frac{n}{\cancel{4}_{1}}\right)+\overset{4}{\cancel{20}}\left(\frac{n}{\cancel{5}_{1}}\right)+20(10\,200)=20n$$

$$10n+5n+4n+204\,000=20n$$

$$19n+204\,000=20n$$

$$19n+204\,000-19n=20n-19n$$

$$204\,000=n$$

$$n=204\,000$$

Verification

LS	RS
$\frac{n}{2}+\frac{n}{4}+\frac{n}{5}+10\,200$	n
$=\frac{204\,000}{2}+\frac{204\,000}{4}+\frac{204\,000}{5}+10\,200$	$=204\,000$
$=102\,000+51\,000+40\,800+10\,200$	
$=204\,000$	

LS = RS

Thus, the total estate was worth \$204 000. Arlene received \$102 000; Brad received \$51 000; and Jill received \$40 800.

3. Let n be the number.

Write the equation and solve.

$$\frac{2}{3}n-\frac{1}{2}=\frac{5}{6}$$

$$6\left(\frac{2}{3}n-\frac{1}{2}\right)=\frac{5}{6}\times 6 \quad \text{(6 is the LCM of 2, 3, and 6.)}$$

$$\overset{2}{\cancel{6}}\left(\frac{2}{\cancel{3}_{1}}n\right)-\overset{3}{\cancel{6}}\left(\frac{1}{\cancel{2}_{1}}\right)=\frac{5}{\cancel{6}_{1}}\times\overset{1}{\cancel{6}}$$

$$4n-3=5$$

$$4n-3+3=5+3$$

$$4n=8$$

$$\frac{4n}{4}=\frac{8}{4}$$

$$n=2$$

Verification

LS	RS
$\frac{2}{3}n-\frac{1}{2}$	$\frac{5}{6}$
$=\frac{2}{3}(2)-\frac{1}{2}$	
$=\frac{4}{3}-\frac{1}{2}$	
$=\frac{8}{6}-\frac{3}{6}$	
$=\frac{5}{6}$	

LS = RS

Therefore, the number is 2.

4. a. $x^2+1=10$

$x^2+1-1=10-1$

$x^2=9$

$x=3$ or -3

b. $r^2-1=24$

$r^2-1+1=24+1$

$r^2=25$

$r=5$ or -5

c. $3t^2=108$

$\frac{3t^2}{3}=\frac{108}{3}$

$t^2=36$

$t=6$ or -6

d. $-2n^2=-128$

$\frac{-2n^2}{-2}=\frac{-128}{-2}$

$n^2=64$

$n=8$ or -8

e. $\sqrt{m}+2=8$

$\sqrt{m}+2-2=8-2$

$\sqrt{m}=6$

$\left(\sqrt{m}\right)^2=6^2$

$m=36$

f. $5\sqrt{n}=10$

$\frac{5\sqrt{n}}{5}=\frac{10}{5}$

$\sqrt{n}=2$

$\left(\sqrt{n}\right)^2=(2)^2$

$n=4$

g. $25x^2=64$

$\frac{25x^2}{25}=\frac{64}{25}$

$x^2=\frac{64}{25}$

$x=\sqrt{\frac{64}{25}}$

$x=\frac{8}{5}$ or $-\frac{8}{5}$

or 1.6 or -1.6

h. $4b^2-3=33$

$4b^2-3+3=33+3$

$4b^2=36$

$\frac{4b^2}{4}=\frac{36}{4}$

$b^2=9$

$b=3$ or -3

5. a. $\sqrt{n}+3=12$

$\sqrt{n}+3-3=12-3$

$\sqrt{n}=9$

$\left(\sqrt{n}\right)^2=(9)^2$

$n=81$

The number is 81.

b. $n^2-8=17$

$n^2-8+8=17+8$

$n^2=25$

$n=5$ or -5

The number is 5 or -5.

c. $10n^2 = 40$

$$\frac{10n^2}{10} = \frac{40}{10}$$

$$n^2 = 4$$

$$n = 2 \text{ or } -2$$

The number is 2 or -2.

d. $3\sqrt{n} = 15$

$$\frac{3\sqrt{n}}{3} = \frac{15}{3}$$

$$\sqrt{n} = 5$$

$$n = 25$$

The number is 25.

6. If $\ell = 82$ and $g = 50$, then

$$m = \frac{\ell \times g^2}{24\,600}$$

$$= \frac{82 \times 50^2}{24\,600}$$

$$= \frac{82 \times 2500}{24\,600}$$

$$= 8.3$$

The mass of the salmon is 8.3 kg.

7. a. $d = \frac{v^2}{210}$

$$= \frac{80^2}{210}$$

$$= \frac{6400}{210}$$

$$\doteq 30.5$$

The breaking distance is about 30.5 m.

b. $d = \frac{v^2}{210}$

$$62 = \frac{v^2}{210}$$

$$62 \times 210 = \frac{v^2}{\cancel{210}_{1}} \times \cancel{210}^{1}$$

$$13\,020 = v^2$$

$$114 \doteq v$$

The speed of the vehicle was about 114 km/h.

8. a. $t = 0.2\sqrt{\ell}$

$$= 0.2\sqrt{64}$$

$$= 0.2 \times 8$$

$$= 1.6$$

The time required for the swing is 1.6 s.

b. $t = 0.2\sqrt{\ell}$

$$2 = 0.2\sqrt{\ell}$$

$$\frac{2}{0.2} = \frac{0.2\sqrt{\ell}}{0.2}$$

$$10 = \sqrt{\ell}$$

$$(10)^2 = \left(\sqrt{\ell}\right)^2$$

$$100 = \ell$$

$$\ell = 100$$

The length of the pendulum would be 100 cm.

Section 2: Activity 1

1. a.
- The maximum speed is 80 km/h.
- Legally, you cannot travel faster than 80 km/h.

b.
- One litre of pop can be purchased for 49¢ if you buy at least 30 L of gas.
- If you buy 30 L (or more) of gasoline, you may purchase 1 L of pop for 49¢.

c.
- Condominiums are being sold for $110 000 and more.
- The starting (or lowest) price for these exclusive condominiums is $110 000.

d.
- The seating capacity of this place is 350 people.
- No more than 350 people are to be allowed in the place.

e.
- The regular price for the movie is $8.00 for adults, $4.50 for students, and $3.00 for children under 12; matinees are half the regular price for all age groups.

f.
- The distance down is 5.3 m.
- The clearance for anything passing under this point is less than 5.3 m.

g.
- Sale in progress with items up to 70% off.
- Some things in the store are on sale for 70% off; other things are reduced less than 70%.

2. The sign only says that the maximum is 80 km/h. You may go any amount less, although you may be asked to speed up if you're going dangerously slow. You may go over, but you are liable to a fine for speeding if caught.

3. There is no maximum, other than your fuel tank's capacity.

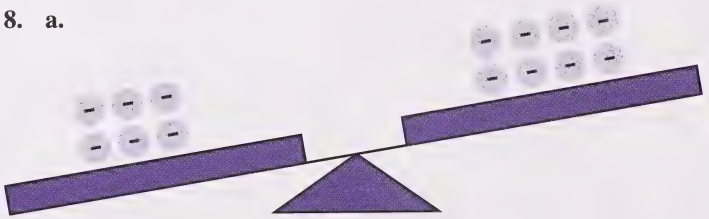
4. a. $4.00 **b.** $2.25 **c.** $1.50

5. No, only one item has to be sold at 70% off. The rest can be sold for less than 70% off.

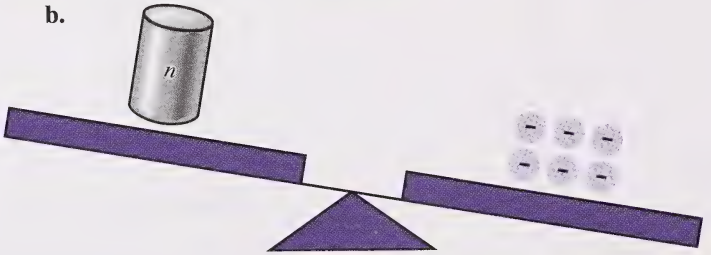
6. The value on the left side is less than the value on the right side.

7. The value on the left side is greater than the value on the right side.

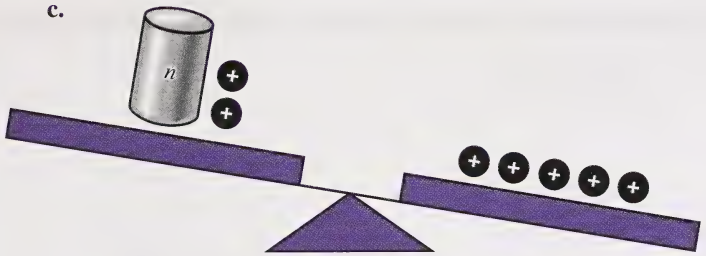
8. a.



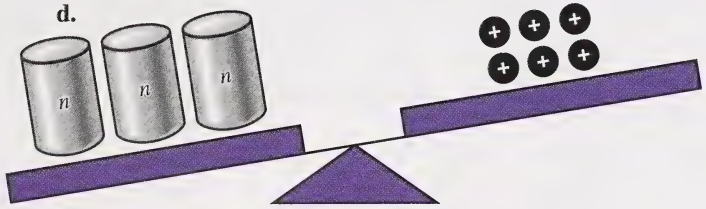
b.



c.



d.



9. **a.** $j \leq 12$ **b.** $d \geq 18$ **c.** $\ell \leq 6$
d. $s < 4$ **e.** $p > 23\,000$ **f.** $m + 2 > 40$
g. $m + 6.50 < 35$ **h.** $3e + 25 > 52$

Section 2: Activity 2

1. **a.** Dots are used because only whole numbers can represent a goal.

b. Yes. Although it is implied in the problem that the player scored some goals, it is possible that Dennis scored no goals in the tournament.

c. No, -1 is not a part of the solution because you cannot score a negative number of goals.

d. No, you cannot score part of a goal.

e. The solution set is all the whole numbers from 0 to 9.

2. **a.** The graph is shown with a solid line drawn to the left of 13 because the solution can be any value less than 13.

b. The value 13 is shown with an open circle because 13 is not part of the solution as indicated by the symbol $<$.

3. **a.** There is a solid line to the right of -6 because the solution can be any value greater than -6.

b. The value -6 is shown with a solid dot because -6 is part of the solution as indicated by the symbol $\geq$.

4. **a.**

$$m + 7 < 10$$
$$m + 7 - 7 < 10 - 7$$
$$m < 3$$

Verification

LS	RS
$m + 7$	10
$= 2 + 7$	
$= 9$	

LS < RS

b. $m-2>11$

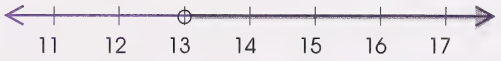
$m-2+2>11+2$

$m>13$

Verification

LS	RS
$m-2$	11
$=14-2$	
$=12$	

LS > RS



c. $x+4\leq 6$

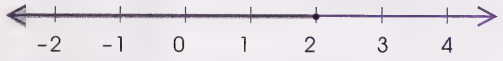
$x+4-4\leq 6-4$

$x\leq 2$

Verification

LS	RS
$x+4$	6
$=1+4$	
$=5$	

LS ≤ RS



d. $y-3>8$

$y-3+3>8+3$

$y>11$

Verification

LS	RS
$y-3$	8
$=12-3$	
$=9$	

LS > RS

e. $w+2>6$

$w+2-2>6-2$

$w>4$

Verification

LS	RS
$w+2$	6
$=5+2$	
$=7$	

LS > RS

f. $n+7\geq -4$

$n+7-7\geq -4-7$

$n\geq -11$

Verification

LS	RS
$n+7$	-4
$=-10+7$	
$=-3$	

LS ≥ RS

5. a.
$$n+7 \le -2$$
$$-7+7 \le -2$$
$$0 \le -2$$

No, $0 \nleq -2$.

b.
$$m-2 > -4$$
$$1-2 > -4$$
$$-1 > -4$$

Yes, $-1 > -4$.

c.
$$w+5 \ge 0$$
$$-5+5 \ge 0$$
$$0 \ge 0$$

Yes, $0 \ge 0$.

d.
$$y-4 < -5$$
$$-4-4 < -5$$
$$-8 < -5$$

Yes, $-8 < -5$.

e.
$$p+4 < 7$$
$$1.5+4 < 7$$
$$5.5 < 7$$

Yes, $5.5 < 7$.

f.
$$t-1 > -2$$
$$-1-1 > -2$$
$$-2 > -2$$

No, $-2 \ngtr -2$.

6. a.
$$c+15 > 110$$
$$c+15-15 > 110-15$$
$$c > 95$$

Jeremy had more than 95 cards before last weekend.

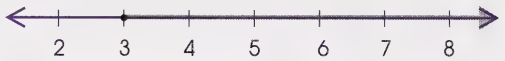
The solution can be displayed on a number line as follows:

b. Although the equation in question 6.a. does not show a maximum, the last sentence implies that he did not have more than 110 cards before last weekend.

7.
$$n+4 \ge 7$$
$$n+4-4 \ge 7-4$$
$$n \ge 3$$

The number is equal to or greater than 3.

The solution can be displayed on a number line as follows:



8.
$$c-35 > 250$$
$$c-35+35 > 250+35$$
$$c > 285$$

Mr. Cartwright had over 285 head of cattle before Wednesday.

The solution can be displayed on a number line as follows:

Section 2: Activity 3

1.

Inequation	Operation to Both Sides	New Inequation	True or False
$-6<2$	Add 2.	$-4<4$	T
$-6<2$	Add -2.	$-8<0$	T
$-6<2$	Subtract 2.	$-8<0$	T
$-6<2$	Subtract -2.	$-4<4$	T
$-6<2$	Multiply by 2.	$-12<4$	T
$-6<2$	Multiply by -2.	$12<-4$	F
$-6<2$	Divide by 2.	$-3<1$	T
$-6<2$	Divide by -2.	$3<-1$	F

2. The new inequation was true after adding and subtracting both 2 and -2, and multiplying and dividing by 2.

3. The new inequation was false after multiplying and dividing by -2.

4. The same thing happens when the inequation $8>6$ is used.

Inequation	Operation to Both Sides	New Inequation	True or False
$8>6$	Add 2.	$10>8$	T
$8>6$	Add -2.	$6>4$	T
$8>6$	Subtract 2.	$6>4$	T
$8>6$	Subtract -2.	$10>8$	T
$8>6$	Multiply by 2.	$16>12$	T
$8>6$	Multiply by -2.	$-16>-12$	F
$8>6$	Divide by 2.	$4>3$	T
$8>6$	Divide by -2.	$-4>-3$	F

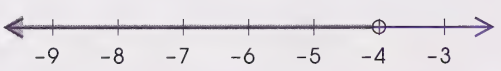
5. You could make the false inequations true by reversing the sign of inequality.

6. When you multiply or divide both sides of an inequation by a negative number, you must reverse the sign of inequality.

7. **a.** No, because when you substitute -4 into the inequation, you get 24 on both sides ($24 \not> 24$).

   b. Yes, because when you substitute -4.1 into the inequation, you get 24.6 on the left side ($24.6 > 24$).

   c.



8. a. Divide by 4.

b. Divide by -2.

c. Divide by 1.5.

9. a. $4m < 12$

$$\frac{4m}{4} < \frac{12}{4}$$

$$m < 3$$

Verification

LS	RS
$4m$	12
$= 4(2)$	
$= 8$	

LS < RS

b. $3m < 27$

$$\frac{3m}{3} < \frac{27}{3}$$

$$m < 9$$

Verification

LS	RS
$3m$	27
$= 3(8)$	
$= 24$	

LS < RS

c. $-5b > 30$

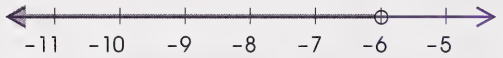
$$\frac{-5b}{-5} < \frac{30}{-5}$$

$$b < -6$$

Verification

LS	RS
$-5b$	30
$= -5 \times (-7)$	
$= 35$	

LS > RS



d. $-4b \geq 10$

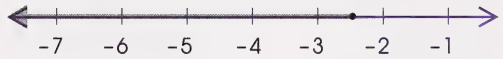
$$\frac{-4b}{-4} \leq \frac{10}{-4}$$

$$b \leq -2.5$$

Verification

LS	RS
$-4b$	10
$= -4 \times (-3)$	
$= 12$	

LS ≥ RS



e. $2.4b < 8.4$

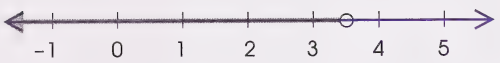
$$\frac{2.4b}{2.4} < \frac{8.4}{2.4}$$

$$b < 3.5$$

Verification

LS	RS
$2.4b$	8.4
$= 2.4(3.4)$	
$= 8.16$	

LS < RS

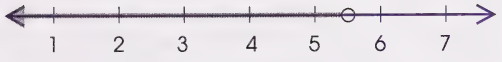


10. a. $8n < 44$

$$\frac{8n}{8} < \frac{44}{8}$$

$$n < 5.5$$

The number is any value less than 5.5.

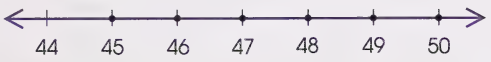


b. $6.25n > 280$

$$\frac{6.25n}{6.25} > \frac{280}{6.25}$$

$$n > 44.8$$

Jason will have to work at least 45 h in the next three weeks.



c. $2.25n > 1500$

$$\frac{2.25n}{2.25} > \frac{1500}{2.25}$$

$$n > 666.\overline{6}$$

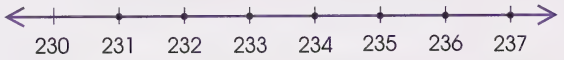
The theatre needs to sell at least 667 tickets.

d. i. Let n be the number of students.

$$3.25n > 750$$

$$\frac{3.25n}{3.25} > \frac{750}{3.25}$$

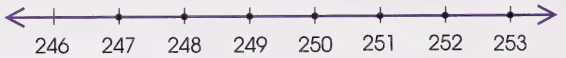
$n > 230.8$ (rounded to the nearest tenth)



They must have at least 231 students at the dance.

ii. $3.25n > 800$

$n > 246.2$ (rounded to the nearest tenth)



They must have at least 247 students at the dance.

11. Helia works 48 weeks of the year.
Let c be the amount she spends each week.

$$48c < 10\,000$$

$$\frac{48c}{48} < \frac{10\,000}{48}$$

$$c < 208.33$$

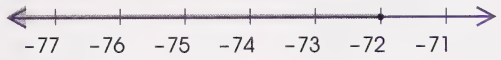
Helia can spend about \$208 per week.

12. a. Yes, $-\frac{72}{6} = -12\,(-12 \le -12)$.

b. Yes, $-\frac{75}{6} = -12.5\,(-12.5 \le -12)$.

c. There are an infinite number of possibilities. Any number less than are equal to -72 is a possible value for a.

d.



13. a. Both sides were multiplied by -3 in order to isolate the variable.

b. The sign of inequality changed because both sides were multiplied by a negative number.

14. a. Multiply by -3.

b. Multiply by 2.

c. Multiply by $\frac{3}{2}$.

15. a.

$$\frac{3}{4}b > \frac{15}{16}$$

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}}b \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}} > \frac{\overset{5}{\cancel{15}}}{\underset{4}{\cancel{16}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}}$$

$$b > \frac{5}{4} \text{ or } 1.25$$

Verification

LS	RS
$\frac{3}{4}b$	$\frac{15}{16}$
$= \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}}\left(\overset{1}{\cancel{2}}\right)$	
$= \frac{3}{2}$ or 1.5	

LS $\ge$ RS

b.

$$-\frac{b}{2} \le -4$$

$$-\frac{b}{2} \times (-2) \ge -4 \times (-2)$$

$$+\left(\frac{b}{\underset{1}{\cancel{2}}} \times \overset{1}{\cancel{2}}\right) \ge +(4 \times 2)$$

$$b \ge 8$$

Verification

LS	RS
$-\frac{b}{2}$	-4
$= -\frac{8}{2}$	
$= -4$	

LS $\le$ RS

16. $\frac{78+82+88+n}{4} > 85$

$$\frac{248+n}{4} > 85$$

$$\frac{248+n}{\cancel{4}_{1}} \times \cancel{4}^{1} > 85 \times 4$$

$$248+n > 340$$

$$248+n-248 > 340-248$$

$$n > 92$$

Alexis must obtain a mark of at least 92% on her fourth test.

17. $\frac{n}{5} < -12$

$$\frac{n}{\cancel{5}_{1}} \times \cancel{5}^{1} < -12 \times 5$$

$$n < -60$$

The largest integer that satisfies this condition is -61.

18. $n - \frac{40}{100}n \leq 85$

$$\frac{100n}{100} - \frac{40n}{100} \leq 85$$

$$\frac{60n}{100} \leq 85$$

$$\frac{60n}{100} \times \frac{100}{60} \leq 85 \times \frac{100}{60}$$

$$n \leq 141.67$$

The highest-priced jacket that Nathan can afford is $141.67.

Now Try This

19. Catastrophic, algebra, level, and remember are all quarks because they start and end with the same letter.

20. **a.** I and W are quimps. K and V are quacks.

b. They are both capital letters made up of line segments.

c. Quimps have some parallel line segments, quacks do not.

d. B, G, O, P, and Q are neither quimps nor quacks because they have some curved lines.

Section 2: Activity 4

1. a. no

b. no

c. Yes, because you have to divide both sides by -3.

d. no

e. no

f. Yes, because you have to divide both sides by -2.5.

2. a. $-7b < 2$

b. $m \leq -3$

c. $m < -32$

d. $-6m > 12$

$m < -2$

e. $-4p \leq -16$

$p \geq 4$

f. $-1.5w > 9$

$w < -6$

3. a.

$$4m - 2 > -6$$
$$4m - 2 + 2 > -6 + 2$$
$$4m > -4$$
$$\frac{4m}{4} > \frac{-4}{4}$$
$$m > -1$$

Verification

LS	RS
$4m-2$	-6
$=4(1)-2$	
$=4-2$	
$=2$	

LS $>$ RS

b.

$$5 - 3d \leq 20$$
$$5 - 3d - 5 \leq 20 - 5$$
$$-3d \leq 15$$
$$\frac{-3d}{-3} \geq \frac{15}{-3}$$
$$d \geq -5$$

Verification

LS	RS
$5-3d$	20
$=5-3(-4)$	
$=5+12$	
$=17$	

LS $\leq$ RS

c.

$$\frac{m}{2} + 1 \leq 3$$
$$\frac{m}{2} + 1 - 1 \leq 3 - 1$$
$$\frac{m}{2} \leq 2$$
$$\frac{m}{\cancel{2}_{1}} \times \cancel{2}^{1} \leq 2 \times 2$$
$$m \leq 4$$

Verification

LS	RS
$\frac{m}{2}+1$	3
$=\frac{2}{2}+1$	
$=1+1$	
$=2$	

LS $\leq$ RS

d.

$$-p + 4 > 3$$
$$-p + 4 - 4 > 3 - 4$$
$$-p > -1$$
$$(-1)(-p) < (-1)(-1)$$
$$p < 1$$

Verification

LS	RS
$-p+4$	3
$=-(-1)+4$	
$=1+4$	
$=5$	

LS $>$ RS

e. $\frac{1}{4}t - 5 < 1$

$\frac{1}{4}t - 5 + 5 < 1 + 5$

$\frac{1}{4}t < 6$

$\frac{1}{\cancel{4}_{1}}t \times \cancel{4}^{1} < 6 \times 4$

$t < 24$

Verification

LS	RS
$\frac{1}{4}t - 5$	1
$= \frac{1}{4}(20) - 5$	
$= 5 - 5$	
$= 0$	

LS < RS

f. $-2.5m + 8 \le -2$

$-2.5m + 8 - 8 \le -2 - 8$

$-2.5m \le -10$

$\frac{-2.5m}{-2.5} \ge \frac{-10}{-2.5}$

$m \ge 4$

Verification

LS	RS
$-2.5m + 8$	-2
$= -2.5(6) + 8$	
$= -15 + 8$	
$= -7$	

LS ≤ RS

4. $3n + 4 < 10$

$3n + 4 - 4 < 10 - 4$

$3n < 6$

$\frac{3n}{3} < \frac{6}{3}$

$n < 2$

The largest integer for which this is true is 1. Two integers for which it is not true are 2 and 3.

5. $11n + 80 > 1000$

$11n + 80 - 80 > 1000 - 80$

$11n > 920$

$\frac{11n}{11} > \frac{920}{11}$

$n > 83.\overline{63}$

Each adult averaged more than $83.\overline{63}\left(83\frac{7}{11}\right)$ kg.

6. $\frac{2n + 84 + 82}{4} \ge 80$ ← Simplify.

$\frac{2n + 166}{4} \ge 80$

$\frac{2n + 166}{\cancel{4}_{1}} \times \cancel{4}^{1} \ge 80 \times 4$ ← Multiply both sides by 4.

$2n + 166 \ge 320$

$2n + 166 - 166 \ge 320 - 166$ ← Subtract 166 from both sides.

$2n \ge 154$

$\frac{2n}{2} \ge \frac{154}{2}$ ← Divide both sides by 2.

$n \ge 77$

The lowest mark she can get in Social Studies and Language Arts is 77%.

7. Let n be the number of cars Ian has. Thus, $n+5$ is the number of cars Michael has.

$$\begin{aligned} 2n+n+5 &\le 14 \\ 3n+5 &\le 14 \\ 3n+5-5 &\le 14-5 \\ 3n &\le 9 \\ \frac{3n}{3} &\le \frac{9}{3} \\ n &\le 3 \end{aligned}$$

Ian has either 1, 2, or 3 cars in his collection. (Ian must have at least one car to have a collection.)

8. Let n be the number of kilometres.

$$\begin{aligned} 1.5n+2.5 &\le 40 \\ 1.5n+2.5-2.5 &\le 40-2.5 \\ 1.5n &\le 37.5 \\ \frac{1.5n}{1.5} &\le \frac{37.5}{1.5} \\ n &\le 25 \end{aligned}$$

Mr. Larouche can travel a maximum of 25 km.

Now Try This

9. a.
$$\begin{aligned} x+x+15 &\le 45 \\ 2x+15 &\le 45 \\ 2x+15-15 &\le 45-15 \\ 2x &\le 30 \\ \frac{2x}{2} &\le \frac{30}{2} \\ x &\le 15 \end{aligned}$$

The solution is $x \le 15$.

b. The triangle is isosceles for whole number values of x from 1 to 14.

c. The triangle is equilateral when $x = 15$.

Section 2: Follow-up Activities

Extra Help

1. a. The model indicates the sign in the inequality has changed by tilting the balance the other way.

b. The inequation shows that the sign in the inequality has changed by reversing the inequality symbol.

2. The basic concept is the same; however, you must change the tilt of the balance when multiplying or dividing by a negative number.

3. a. $m+7<10$

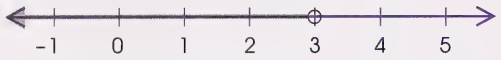
$m+7-7<10-7$

$m<3$

Verification

LS	RS
$m+7$	10
$=2+7$	
$=9$	

LS < RS



b. $m-1>10$

$m-1+1>10+1$

$m>11$

Verification

LS	RS
$m-7$	10
$=11.2-1$	
$=10.2$	

LS > RS

c. $-4m>12$

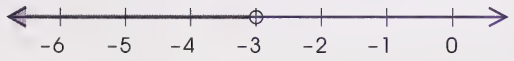
$\frac{-4m}{-4}<\frac{12}{-4}$

$m<-3$

Verification

LS	RS
$-4m$	12
$=-4(-3.5)$	
$=14$	

LS > RS



d. $-4x+8\le 14$

$-4x+8-8\le 14-8$

$-4x\le 6$

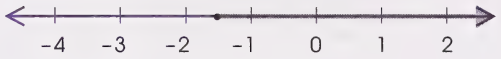
$\frac{-4x}{-4}\ge\frac{6}{-4}$

$x\ge -1.5$

Verification

LS	RS
$-4x+8$	14
$=-4\times(-1)+8$	
$=4+8$	
$=12$	

LS ≤ RS



e. $5m+8<-2$

$5m+8-8<-2-8$

$5m<-10$

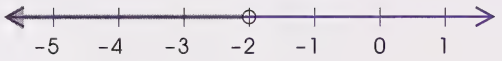
$\frac{5m}{5}<\frac{-10}{5}$

$m<-2$

Verification

LS	RS
$5m+8$	-2
$=5(-3)+8$	
$=-15+8$	
$=-7$	

LS < RS



f. $-0.75p-1<-4$

$-0.75p-1+1<-4+1$

$-0.75p<-3$

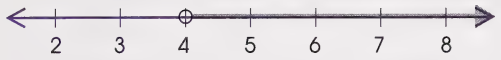
$\frac{-0.75p}{-0.75p}>\frac{-3}{-0.75p}$

$p>4$

Verification

LS	RS
$-0.75p-1$	-4
$=-0.75(8)-1$	
$=-6-1$	
$=-7$	

LS < RS



b. $3d+8>5d-6$

$3d+8-8>5d-6-8$

$3d>5d-14$

$3d-5d>5d-14-5d$

$-2d>-14$

$\frac{-2d}{-2}<\frac{-14}{-2}$

$d<7$

Verification

LS	RS
$3d+8$	$5d-6$
$=3(6)+8$	$=5(6)-6$
$=18+8$	$=30-6$
$=26$	$=24$

LS > RS

Enrichment

1. a. $4(c-2)<12$

$4c-8<12$

$4c-8+8<12+8$

$4c<20$

$\frac{4c}{4}<\frac{20}{4}$

$c<5$

Verification

LS	RS
$4(c-2)$	12
$=4(4-2)$	
$=4(2)$	
$=8$	

LS < RS

c. $7e+9\le 5e-6$

$7e+9-9\le 5e-6-9$

$7e\le 5e-15$

$7e-5e\le 5e-15-5e$

$2e\le -15$

$\frac{2e}{2}\le\frac{-15}{2}$

$e\le -7.5$

Verification

LS	RS
$7e+9$	$5e-6$
$=7(-8)+9$	$=5(-8)-6$
$=-56+9$	$=-40-6$
$=-47$	$=-46$

LS ≤ RS

d.

$$9-2f<4f-15$$
$$9-2f-9<4f-15-9$$
$$-2f<4f-24$$
$$-2f-4f<4f-24-4f$$
$$-6f<-24$$
$$\frac{-6f}{-6}>\frac{-24}{-6}$$
$$f>4$$

Verification

LS	RS
$9-2f$	$4f-15$
$=9-2(5)$	$=4(5)-15$
$=9-10$	$=20-15$
$=-1$	$=5$

LS < RS

e.

$$4g\geq 9g-35$$
$$4g-9g\geq 9g-35-9g$$
$$-5g\geq -35$$
$$\frac{-5g}{-5}\leq\frac{-35}{-5}$$
$$g\leq 7$$

Verification

LS	RS
$4g$	$9g-35$
$=4(6)$	$=9(6)-35$
$=24$	$=54-35$
	$=19$

LS ≥ RS

f.

$$6(n+2)>36$$
$$6n+12>36$$
$$6n+12-12>36-12$$
$$6n>24$$
$$\frac{6n}{6}>\frac{24}{6}$$
$$n>4$$

Verification

LS	RS
$6(n+2)$	36
$=6(5+2)$	
$=6(7)$	
$=42$	

LS > RS

2. An irrational number is one which cannot be expressed as an integer or a quotient of integers. Irrational numbers include numbers like $\sqrt{2}$, $\sqrt{5}$, and π.

Real numbers include both rational numbers (which you have studied in this course) and irrational numbers.

Complex numbers include both rational numbers (which you have studied in this course) and irrational numbers. Complex numbers include all the real numbers as well as a group of numbers called imaginary numbers. Imaginary numbers are numbers that are written in the form $a+bi$, where a and b are real numbers and i is an imaginary number defined as $\sqrt{-1}$. Complex numbers are useful in higher mathematics in the study of coordinate planes in spaces beyond the normal three dimensions.

Puzzles

How Did the Doe Win the Big Animal Race?

Solve each problem. Cross out the box that contains the answer. When you finish, write the letters from the remaining boxes in the squares at the bottom of the page.

1. Mr. Merrill has three times as many nickels as dimes. The coins have a total value of $1.50. How many of each coin does he have?
   _____ nickels, _____ dimes

2. Ms. Lynch has 21 coins in nickels and dimes. Their total value is $1.65. How many of each coin does she have?
   _____ nickels, _____ dimes

3. A vending machine that takes only dimes and quarters contains 30 coins, with a total value of $4.20. How many of each coin are there?
   _____ dimes, _____ quarters

4. The total value of the $1 bills and $5 bills in a cash box is $124. There are eight more $5 bills than $1 bills. How many of each are there?
   _____ $1 bills, _____ $5 bills

5. A collection of nickels and quarters amounts to $2.60. There are 16 coins in all. How many of each coin are there?
   _____ nickels, _____ quarters

6. Joe Lick bought some 20¢ and 25¢ stamps. He bought 32 stamps in all, and paid $7.40 for them. How many stamps of each kind did he buy?
   _____ 20¢, _____ 25¢

7. For a school play, 340 tickets valued at $810 were sold. Some cost $2 and some cost $3. How many tickets of each kind were sold?
   _____ $2 tickets, _____ $3 tickets

8. Romeo bought a mixture of 20¢, 35¢, and 50¢ valentines. The number of 20¢ valentines was one more than twice the number of 35¢ valentines, and the number of 50¢ valentines was two less than the number of 35¢ ones. If he spent $4.20 all together, how many valentines of each kind did he buy?
   _____ 20¢, _____ 35¢, _____ 50¢

IT	SH	ER	EW	EP	ON	AN	AS
22, 8	6, 10	7, 9	9, 4, 2	13, 6, 4	18, 6	14, 22	240, 100
SE	DT	EN	HE	BU	KI	CK	LL
19, 11	13, 21	210, 130	8, 13	200, 140	12, 20	11, 21	9, 12

SOURCE: © 1989 CREATIVE PUBLICATIONS, MOUNTAIN VIEW, CA

Books Never Written

Tragedy on the Cliff by _____

4 −7 3 4 4 −11 6 12 −2 4 −5

Mystery of the Creaking Door by _____

−5 −15 10 −9 1 8 −7 −11 −4 4 10

P.S. by _____

−1 6 −1 3 −7 −11 4 2 12 12 −5 4

Above are the titles of three "Books Never Written." To decode the names of their authors, follow these directions.

Solve any problem below and find the answer in the code at the top of the page. Each time the answer appears, write the letter of that problem above it. Keep working and you will decode the names of all three authors.

(L) Eight less than 7 times a number is the same as 4 more than 3 times the number. Find the number.

(V) Four more than 6 times a number is the same as 9 times the number increased by 10. Find the number.

(I) Four times a number is the same as 14 less than twice the number. Find the number.

(Y) One more than 8 times a number is the same as 12 times the number decreased by 3. Find the number.

(G) Twelve less than a number is the same as 5 times the number increased by 4. Find the number.

(D) Twenty decreased by 2 times a number is the same as 10 less than 3 times the number. Find the number.

(M) Two more than a number is the same as 16 decreased by 6 times the number. Find the number.

(R) Four times a number decreased by 25 is the same as 9 times the number. Find the number.

(T) Twice a number plus 6 times the number is −72. Find the number.

(H) A number plus 5 more than 3 times the number is 37. Find the number.

(A) Eleven diminished by 5 times a number is the same as 4 times the number increased by 20. Find the number.

(S) Eight times a number is the same as 90 decreased by the number. Find the number.

(U) Nine more than 3 times a number is the same as 6 less than twice the number. Find the number.

(O) One increased by 7 times a number is the same as 5 times the number plus 25. Find the number.

(E) Twenty-eight decreased by 6 times a number is the same as the number. Find the number.

(N) Sixteen diminished by 8 times a number is the same as 5 diminished by 9 times the number. Find the number.

SOURCE: © 1982 CREATIVE PUBLICATIONS, MOUNTAIN VIEW, CA

What Did They Call the Bug that the Astronauts Brought Back from the Moon?

Solve each problem. Find your solution at the bottom of the page and cross out the letter above it. When you finish, the answer to the title question will remain.

(1) Andy is twice as old as Kate. In six years, their ages will total 60. How old is each now?

Kate _____, Andy _____

(2) Mrs. Wang is 23 years older than her daughter. In 5 years, their ages will total 63. How old are they now?

daughter _____, Mrs. Wang _____

(3) Matthew is three times as old as Jenny. In seven years, he will be twice as old as she will be then. How old is each now?

Jenny _____, Matthew _____

(4) Juan is eight years older than his sister. In three years, he will be twice as old as she will be then. How old are they now?

sister _____, Juan _____

(5) Melissa is 24 years younger than Joyce. In 2 years, Joyce will be three times as old as Melissa will be then. How old are they now?

Joyce _____, Melissa _____

(6) Tom is four years older than Jerry. Nine years ago, Tom was five times as old as Jerry was then. How old is each now?

Jerry _____, Tom _____

(7) Kathy is six years younger than Bill. Twelve years ago, Bill was twice as old as Kathy was then. How old are they now?

Bill _____, Kathy _____

(8) Dr. Garcia is twice as old as his son. Twenty years ago, he was four times as old as his son was then. How old are they now?

son _____, Dr. Garcia _____

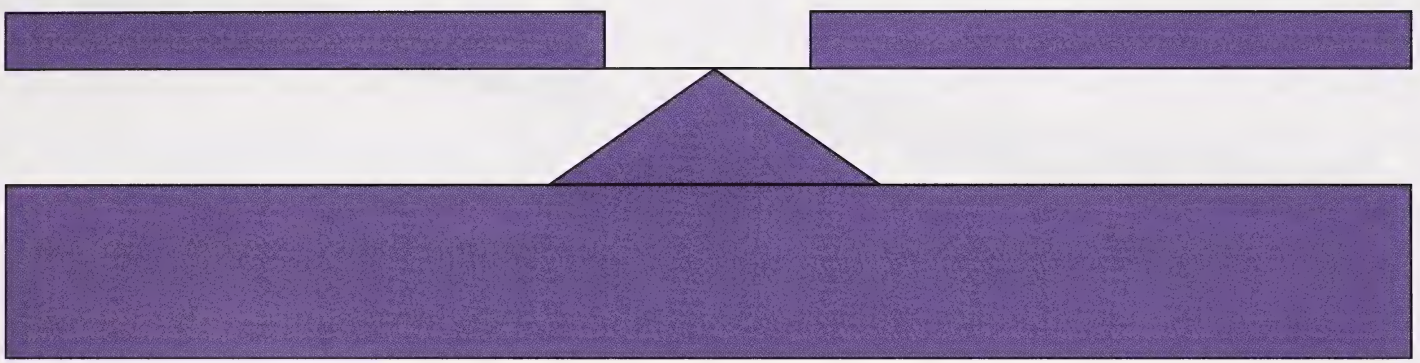
B	A	F	L	E	Y	U	G	N	A	T	O	I	T	C	H	K
5, 13	7, 15	30, 60	14, 28	16, 32	10, 14	33, 66	7, 21	26, 20	9, 32	35, 11	24, 18	8, 24	34, 10	8, 12	15, 38	21, 16

SOURCE: © 1989 CREATIVE PUBLICATIONS, MOUNTAIN VIEW, CA

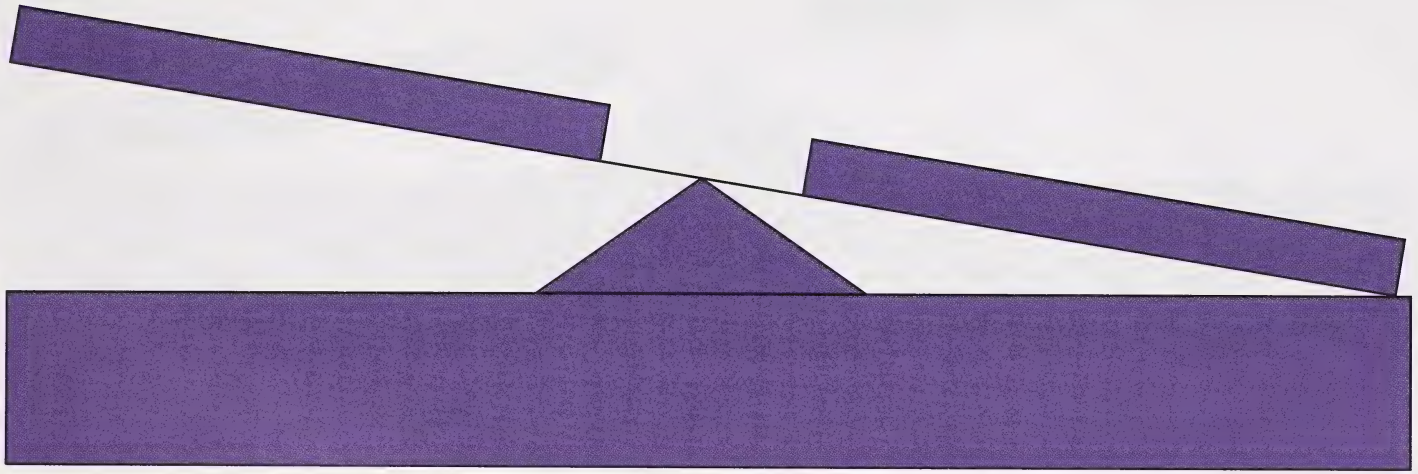
Cut-out Learning Aids

You may wish to glue the cut-out learning aids to heavier paper before you cut them out.

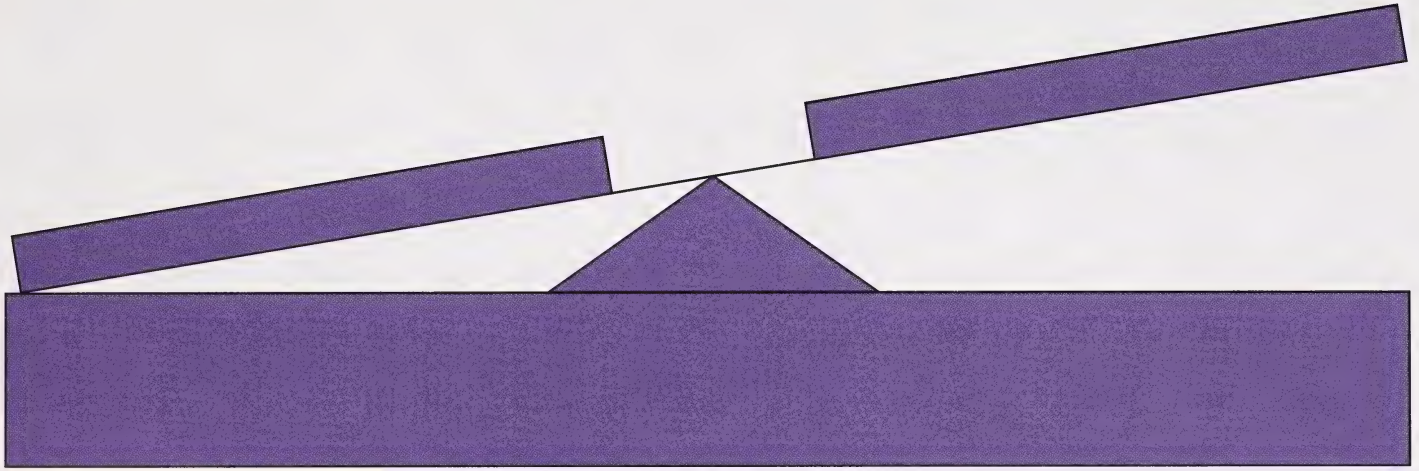
Equation Scale



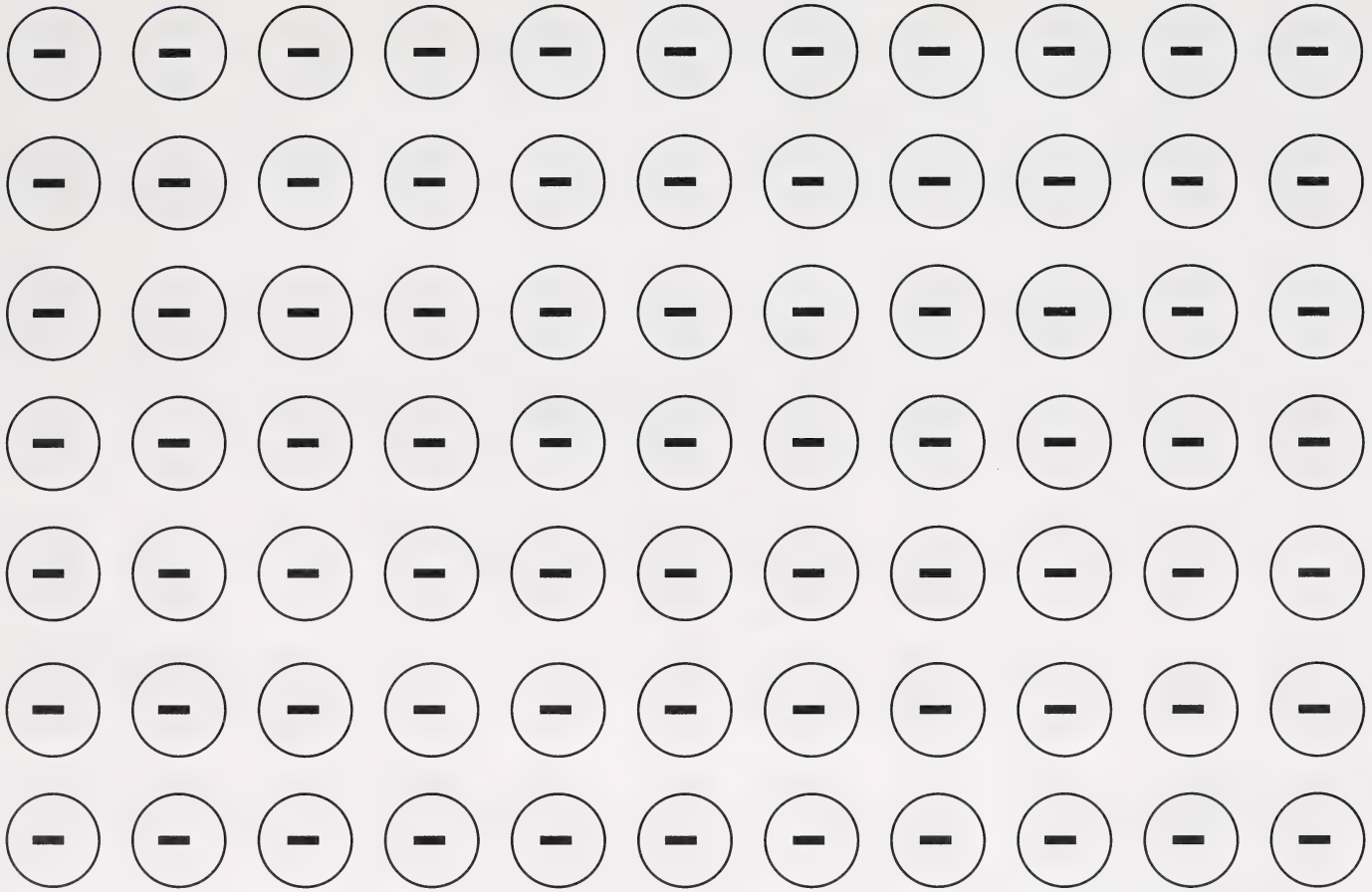
Inequation Scale A



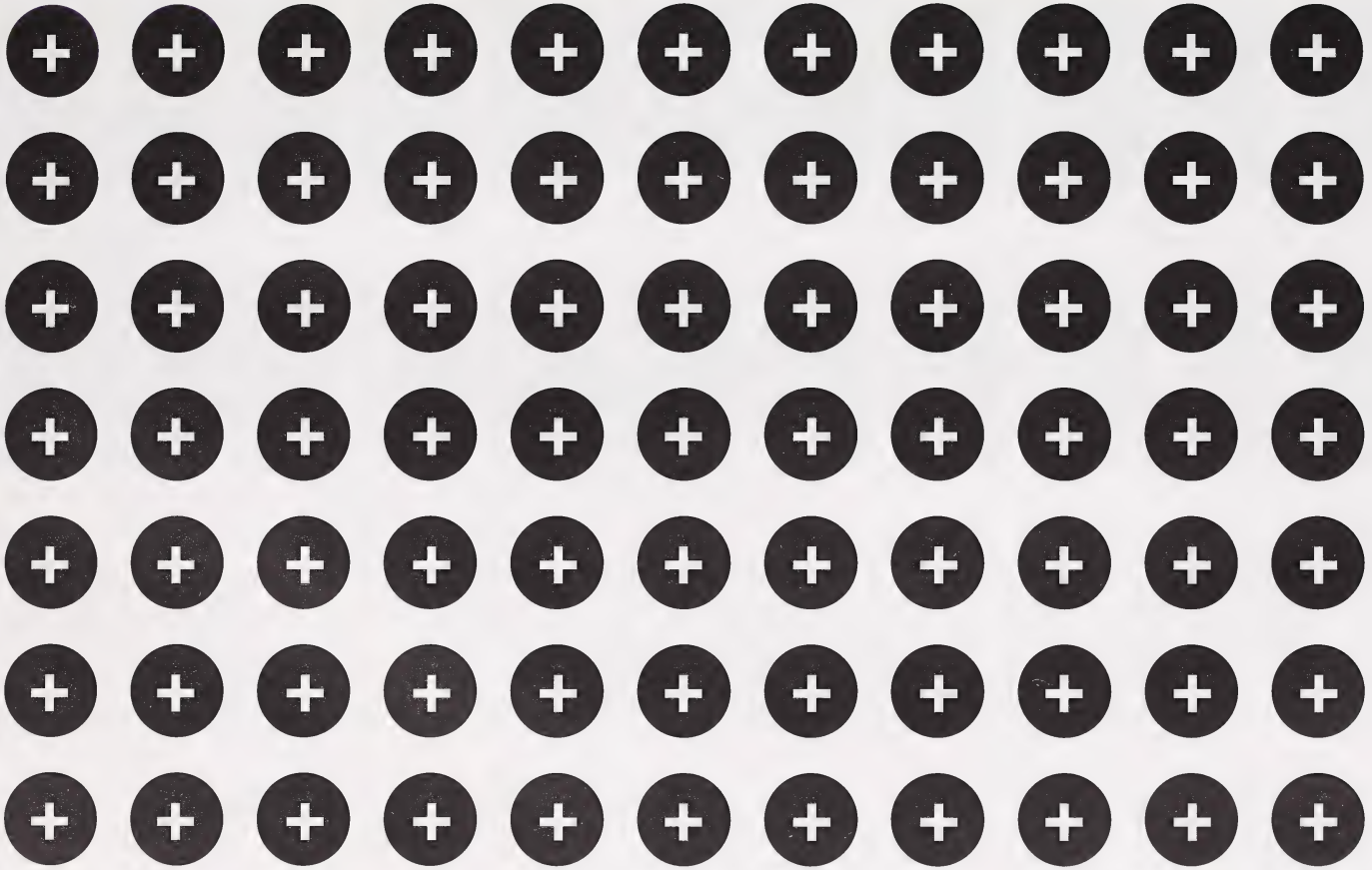
Inequation Scale B



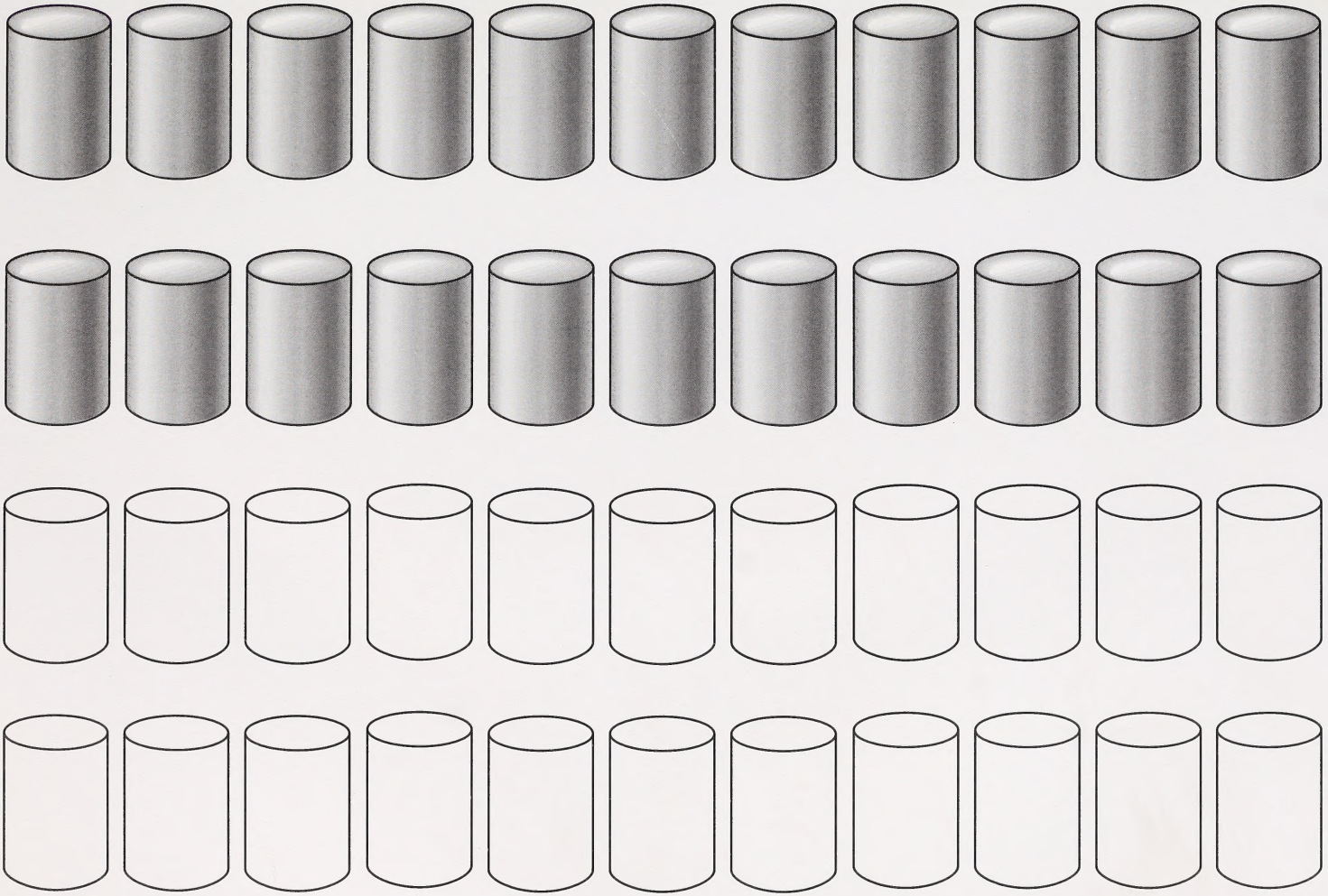
Negative Counters

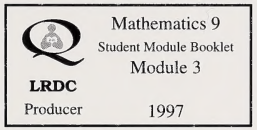


Positive Counters



Cylinders




Mathematics 9
Student Module Booklet
Module 3
LRDC
Producer
1997